An introduction to generalised hydrodynamics in quantum integrable systems

Maurizio Fagotti

11 September 2018

Non-equilibrium behaviour of isolated classical and quantum systems, SISSA, Trieste

Summary

- Nonequilibrium dynamics in isolated many-body systems
- Relevance of inhomogeneities
- From exact time evolution to hydrodynamic descriptions
- Generalised hydrodynamics in interacting integrable systems

Quench dynamics

- the system time evolves unitarily

$$\left|\Psi(t)\right\rangle = e^{-i\hat{H}t}\left|\Psi(0)\right\rangle \quad (\hat{\rho} = \left|\Psi\right\rangle\left\langle\Psi\right|)$$

$$\hat{\rho}(t) = e^{-i\hat{H}t}\hat{\rho}(0)e^{i\hat{H}t}$$

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- subsystems… $\hat{\rho}_A = \mathrm{Tr}_B[\hat{\rho}]$

$$\left\langle \hat{\mathcal{O}} \right\rangle = \mathrm{Tr}_A[\hat{\mathcal{O}}_A\hat{\rho}_A]$$

observable in A

$$\hat{\mathcal{O}} = \hat{\mathcal{O}}_A \otimes \hat{\mathrm{I}}_B$$

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- … don't! $\hat{H} = \hat{H}_A \otimes \hat{\mathrm{I}}_B + \hat{\mathrm{I}}_A \otimes \hat{H}_B + \hat{H}_{AB}$

$$i\partial_t \hat{\rho}_A(t) = [\hat{H}_A, \hat{\rho}_A(t)] + \boxed{\dots}$$

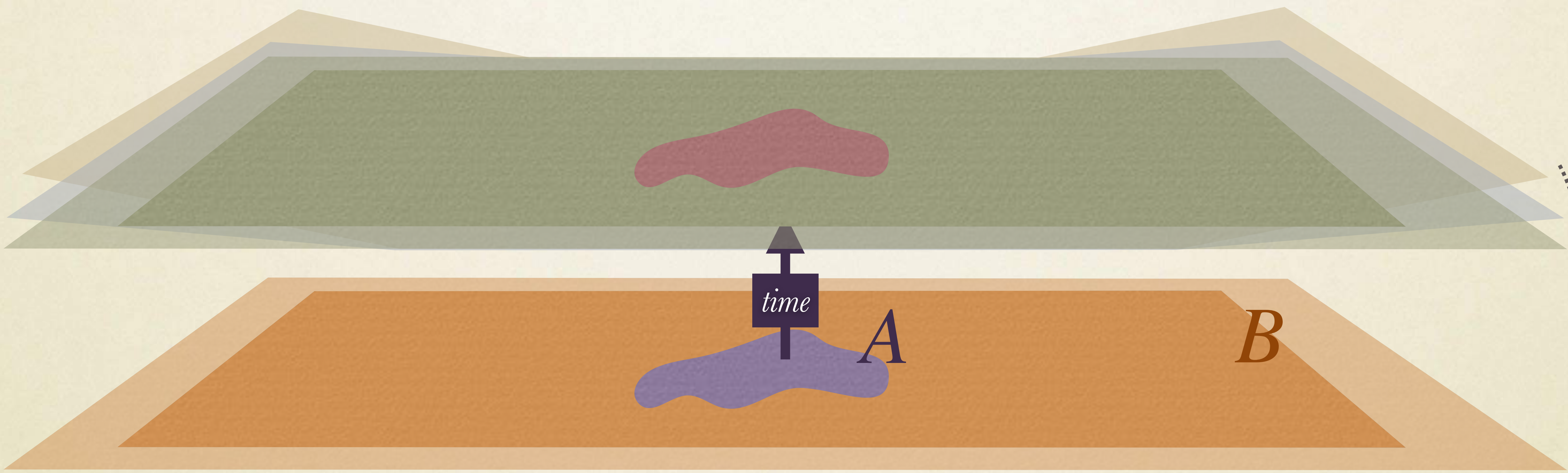
local relaxation is possible!

Quench dynamics
local relaxation

$$\exists \lim_{t\to\infty} \lim_{|B|\to\infty} \hat{\rho}_A(t) = \mathrm{Tr}_B[\hat{\rho}^{MS}]$$

density matrix of a stationary state representing a **_macro-state_**

"underdetermined state"
only the local properties are fixed



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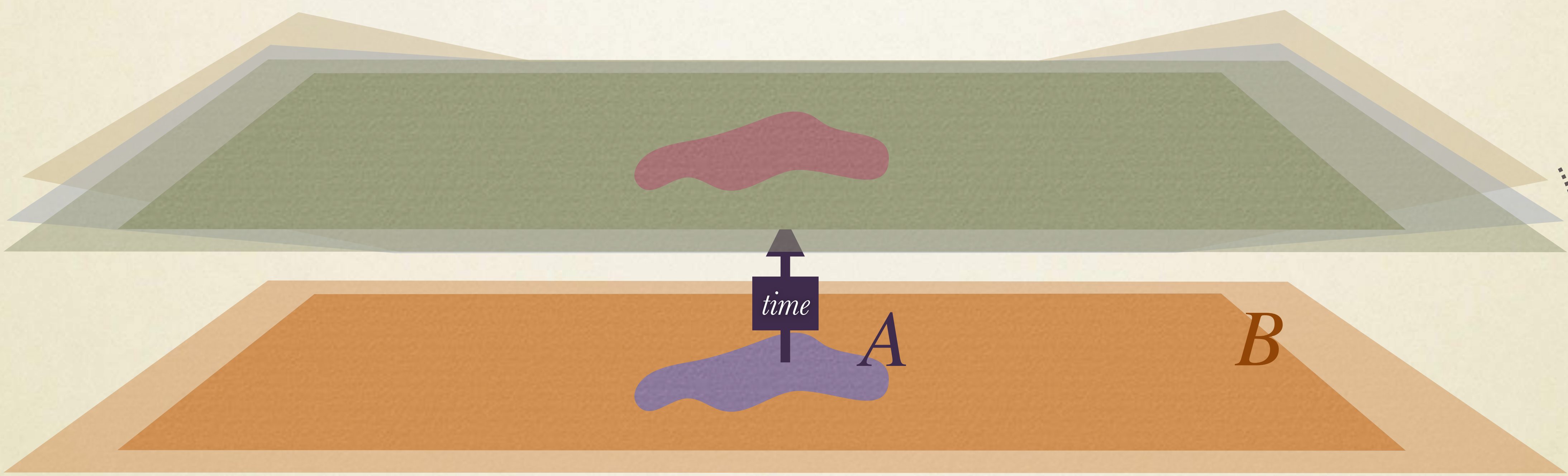

Claim: in a macro-state of a ***translationally invariant*** system with
- (quasi-)local Hamiltonian
- cluster decomposition property

the local properties are fixed by the ***(quasi-)local integrals of motion***

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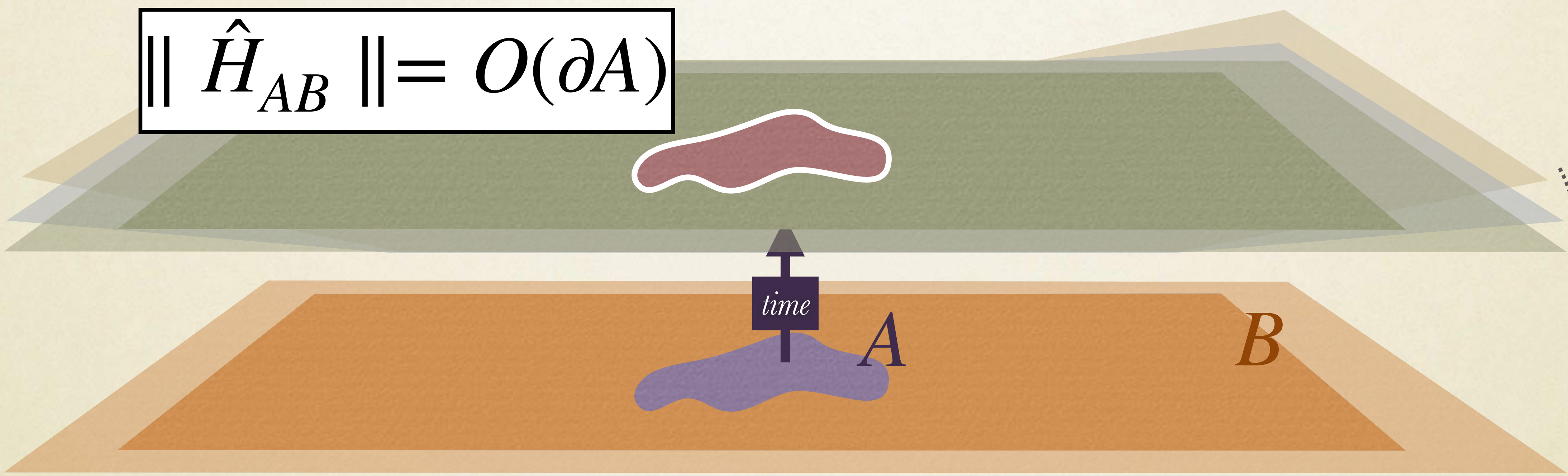
$$\boxed{\| \hat{H}_{AB} \| = O(\partial A)}$$



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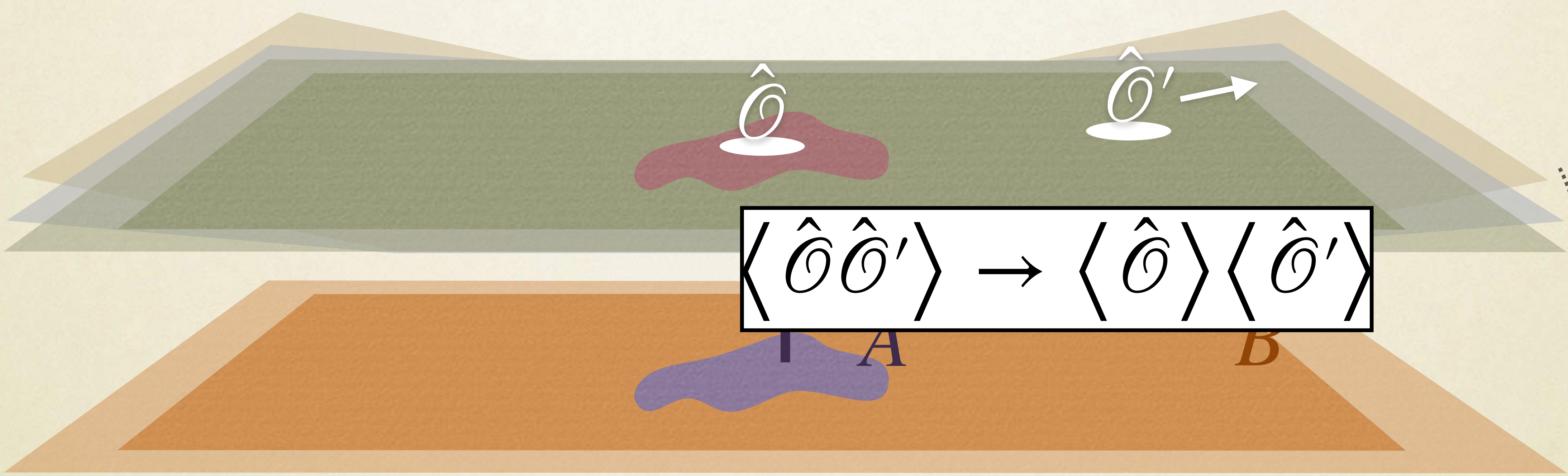

$$\left\langle \hat{\mathcal{O}}\hat{\mathcal{O}}' \right\rangle \to \left\langle \hat{\mathcal{O}} \right\rangle \left\langle \hat{\mathcal{O}}' \right\rangle$$

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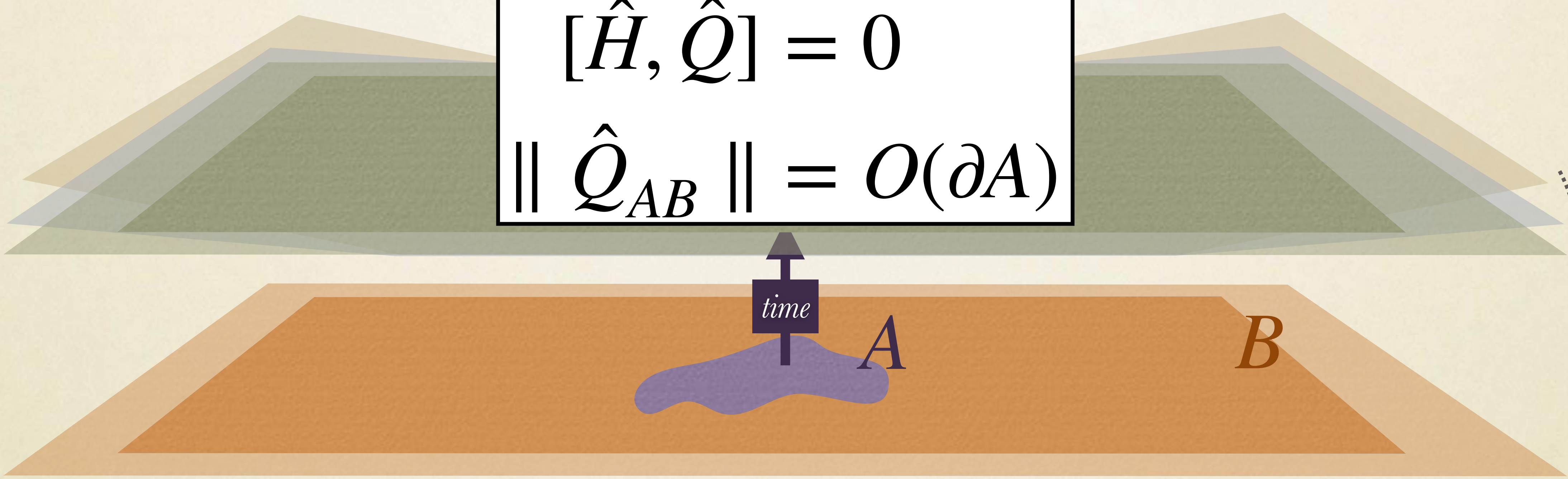
$$[\hat{H}, \hat{Q}] = 0$$

$$\| \hat{Q}_{AB} \| = O(\partial A)$$



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Quench dynamics

local relaxation

CANONICAL PERSPECTIVE

macro-state represented by the least "informative" stationary state

$$\delta\Big[-\mathrm{tr}[\hat{\rho}\log\hat{\rho}] - \sum_{\hat{Q}} \lambda_{\hat{Q}} \mathrm{tr}[\hat{\rho}\hat{Q}]\Big] = 0$$

entropy *(quasi)local integrals of motion*

Quench dynamics
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entropy *(quasi)local integrals of motion*

$$\hat\rho^{MS} = \frac{e^{-\hat Q}}{Z} \qquad [\hat H, \hat Q] = 0$$

- Generic systems: $\hat Q \propto \hat H$, **thermalization**
- Integrable systems: relaxation to a **generalized Gibbs ensemble**

SPECIAL ISSUE ON QUANTUM INTEGRABILITY IN OUT OF EQUILIBRIUM SYSTEMS

Quench dynamics and relaxation in isolated integrable quantum spin chains

To cite this article: Fabian H L Essler and Maurizio Fagotti *J. Stat. Mech.* (2016) 064002

Relevance of inhomogeneities

arXiv:1508.04401 (Aug 2015)

Control of global properties in a closed many-body quantum system by means of a local switch

Maurizio Fagotti
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We consider non-equilibrium time evolution after a quench of a global Hamiltonian parameter in systems described by Hamiltonians with local interactions. Within this background, we propose a protocol that allows to change global properties of the state by flipping a switch that modifies a local term of the Hamiltonian (creating a defect). A light-cone that separates two globally different regions originates from the switch. The expectation values of macroscopic observables, that is to say local observables that are spatially averaged within a subsystem, slowly approach new asymptotic values determined by the defect. The process is almost reversible: flipping again the switch produces a new light-cone with the two regions inverted. Finally, we test the protocol under repeated projective measurements. As explicit example we study the dynamics in a simple exactly solvable model but analogues descriptions apply also to generic models.

- observation of the relevance of inhomogeneities at late times after a quantum quench
- physical explanation

PRL **117,** 130402 (2016) PHYSICAL REVIEW LETTERS week ending 23 SEPTEMBER 2016

Determination of the Nonequilibrium Steady State Emerging from a Defect

Bruno Bertini[1] and Maurizio Fagotti[2]
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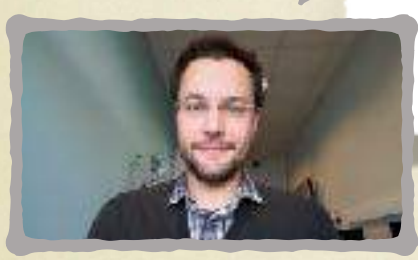
[2] *Département de Physique, École Normale Supérieure / PSL Research University, CNRS, 24 rue Lhomond, 75005 Paris, France*
(Received 26 April 2016; published 22 September 2016)

We consider the nonequilibrium time evolution of a translationally invariant state under a Hamiltonian with a localized defect. We discern the situations where a light cone spreads out from the defect and separates the system into regions with macroscopically different properties. We identify the light cone and propose a procedure to obtain a (quasi)stationary state describing the late time dynamics of local observables. As an explicit example, we study the time evolution generated by the Hamiltonian of the transverse-field Ising chain with a local defect that cuts the interaction between two sites (a quench of the boundary conditions alongside a global quench). We solve the dynamics exactly and show that the late time properties can be obtained with the general method proposed.



- determination of the macro-state
- main (rough) ideas at the base of ***generalised hydrodynamics*** in interacting integrable systems

IOP Publishing Journal of Physics A: Mathematical and Theoretical
J. Phys. A: Math. Theor. **50** (2017) 034005 (34pp) doi:10.1088/1751-8121/50/3/034005

Charges and currents in quantum spin chains: late-time dynamics and spontaneous currents

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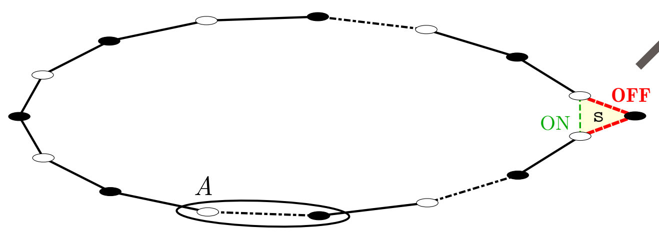
CrossMark

Abstract
We review the structure of the conservation laws in noninteracting spin chains and unveil a formal expression for the corresponding currents. We briefly discuss how interactions affect the picture. In the second part, we explore the effects of a localized defect. We show that the emergence of spontaneous currents near the defect undermines any description of the late-time dynamics by means of a stationary state in a finite chain. In particular, the diagonal ensemble does not work. Finally, we provide numerical evidence that simple generic localized defects are not sufficient to induce thermalization.

- review
- operatorial expression for the currents in noninteracting systems

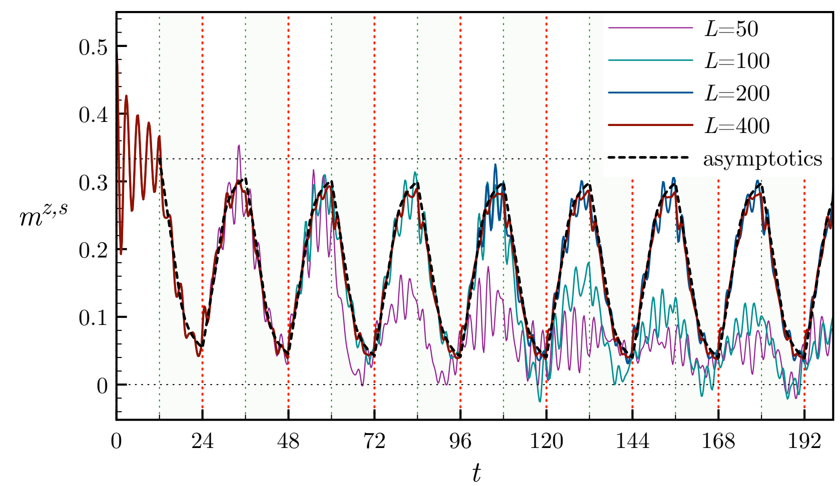
Relevance of inhomogeneities

switch isolating one site



$$\hat{H} = \sum_{\ell=1}^{L} \frac{1}{2}(\hat{c}_\ell \hat{c}^\dagger_{\ell+1} - \hat{c}^\dagger_\ell \hat{c}_{\ell+1}) + \frac{\gamma}{2}(\hat{c}_\ell \hat{c}_{\ell+1} - \hat{c}^\dagger_\ell \hat{c}^\dagger_{\ell+1})$$

with PBC



6 neighbouring spins from the switch

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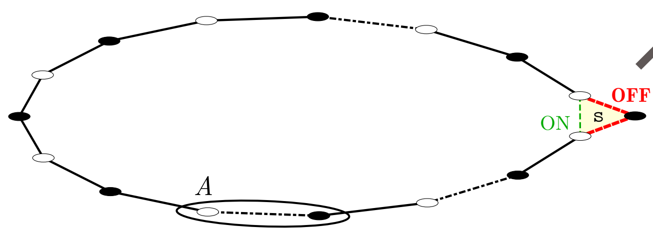

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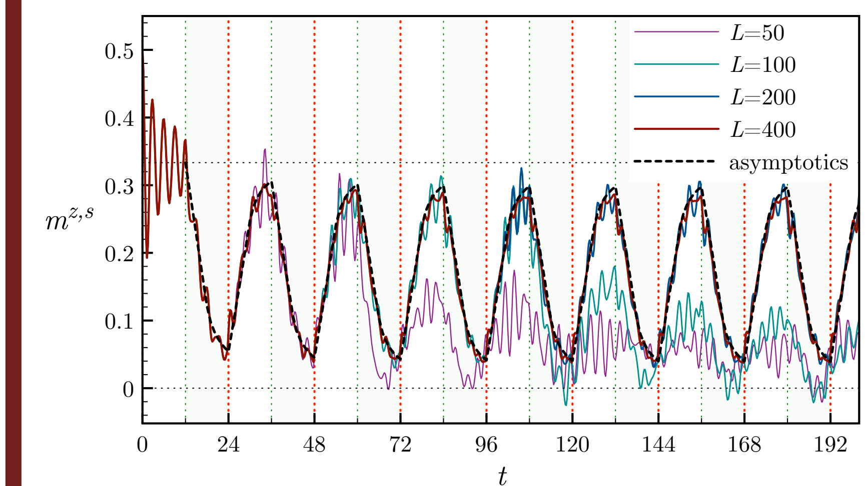

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macroscopic effects lasting infinitely long times at infinitely large distances from the switch

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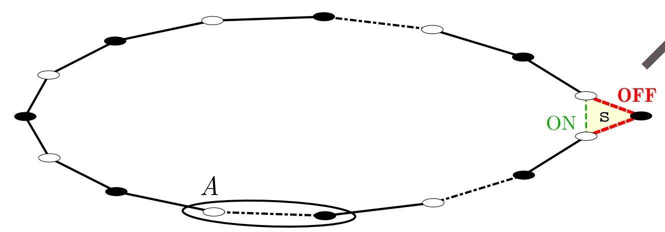

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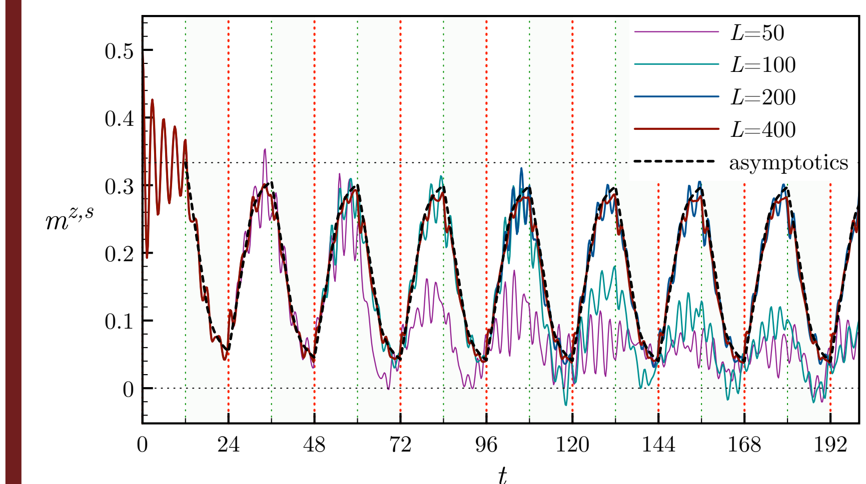

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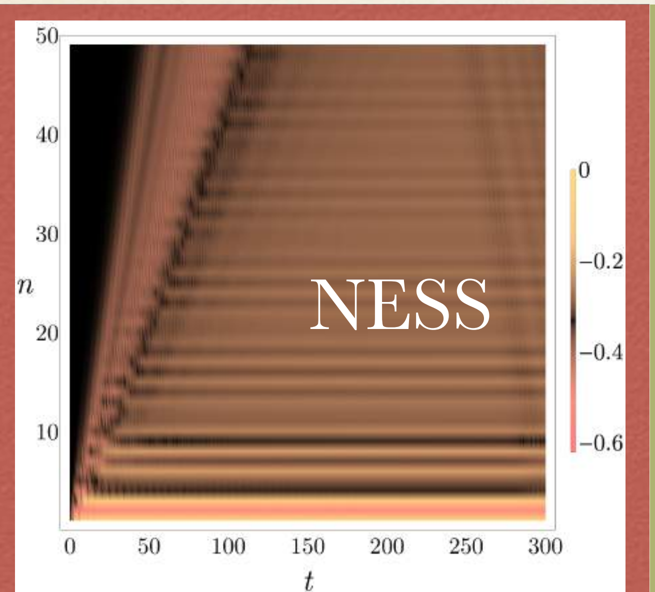
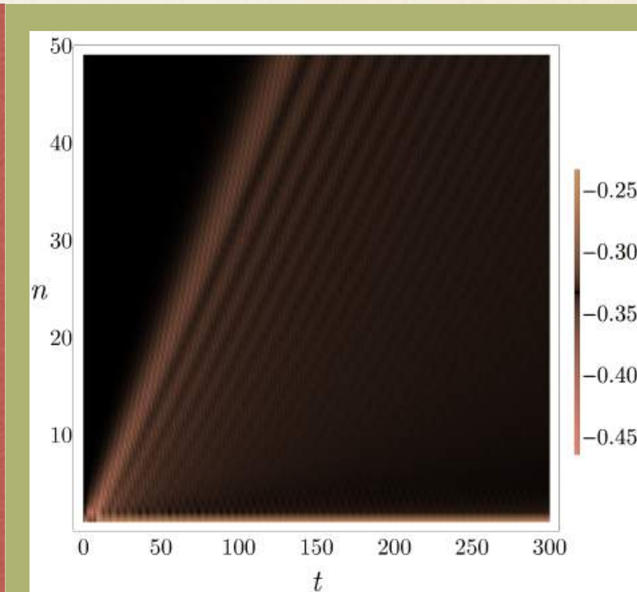
relevant defect





irrelevant defect

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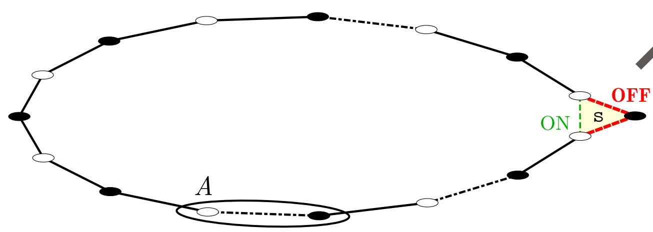

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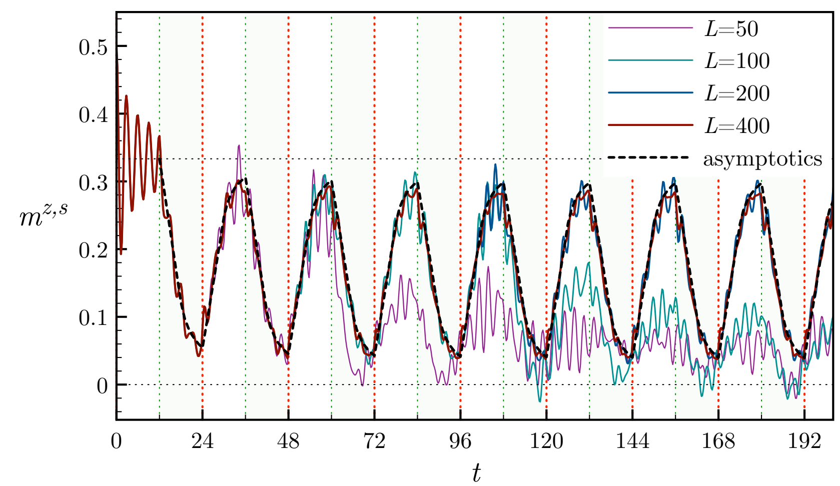

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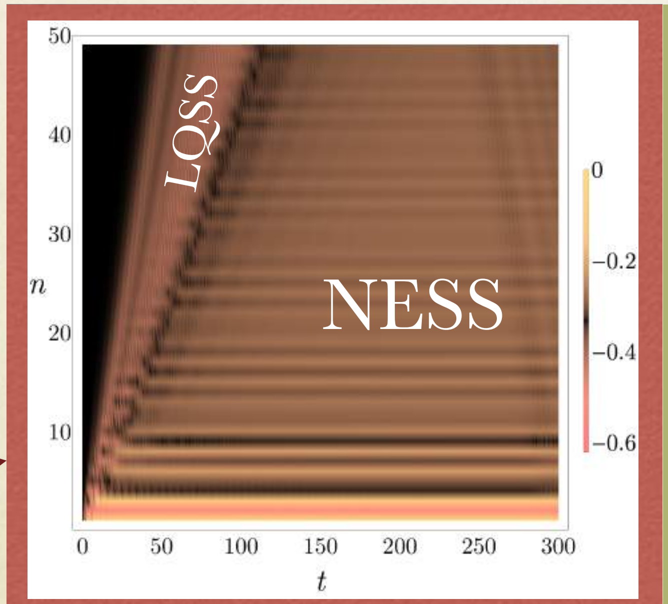

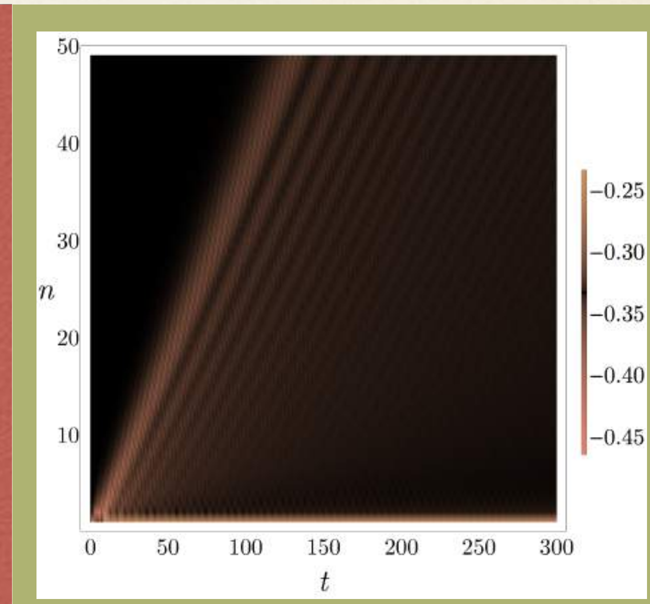

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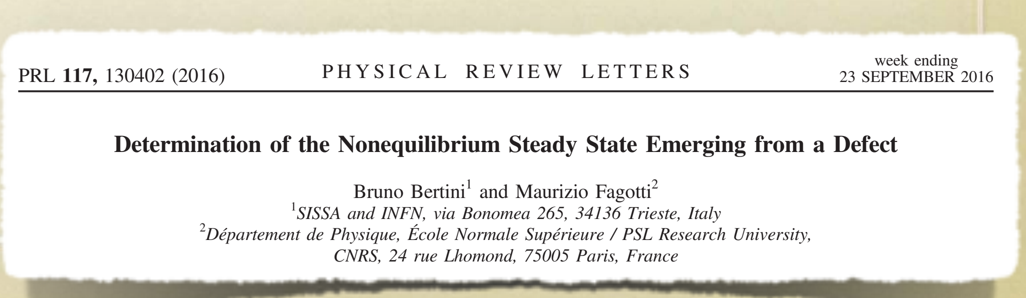
Relevance of inhomogeneities
locally quasistationary state

- In the presence of a relevant local defect, the *space-time scaling limit* $t \to \infty$ at a distance $\ell = \zeta t + o(t)$ is described by a ζ-dependent stationary state for the clean model (for $\zeta \neq 0$)

proved in the quantum Ising model with a defect cutting the interaction between two sites


PRL 117, 130402 (2016) PHYSICAL REVIEW LETTERS week ending 23 SEPTEMBER 2016

Determination of the Nonequilibrium Steady State Emerging from a Defect

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- In the presence of a relevant local defect, the *space-time scaling limit* $t \to \infty$ at a distance $\ell = \zeta t + o(t)$ is described by a ζ-dependent stationary state for the clean model (for $\zeta \neq 0$)

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- Outside a light cone, the defect becomes irrelevant $\longrightarrow$ **Lieb-Robinson velocity**

Commun. math. Phys. 28, 251—257 (1972)

The Finite Group Velocity of Quantum Spin Systems

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Cambridge, Massachusetts, USA

Derek W. Robinson**
Dept. of Physics, Univ. Aix-Marseille II, Marseille-Luminy, France

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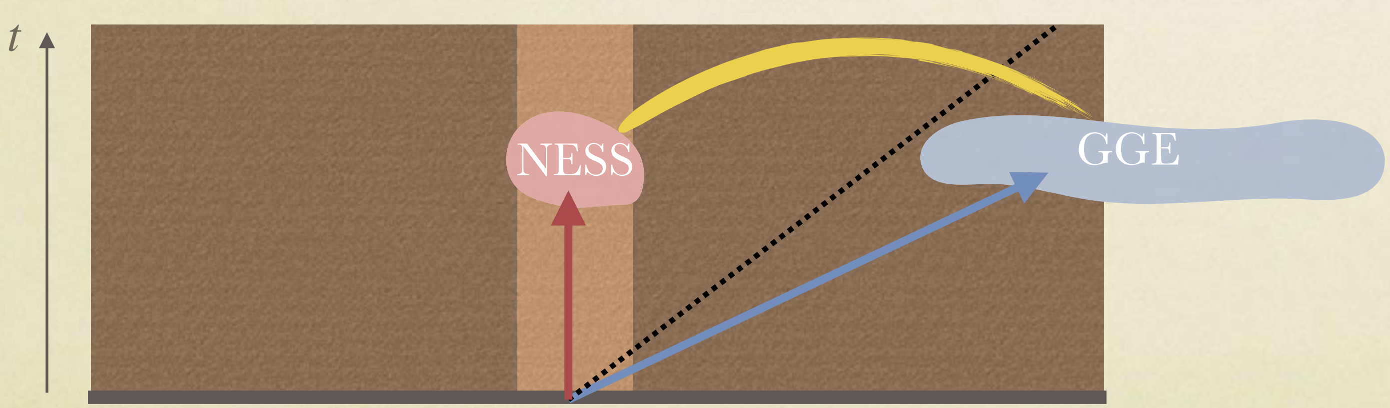
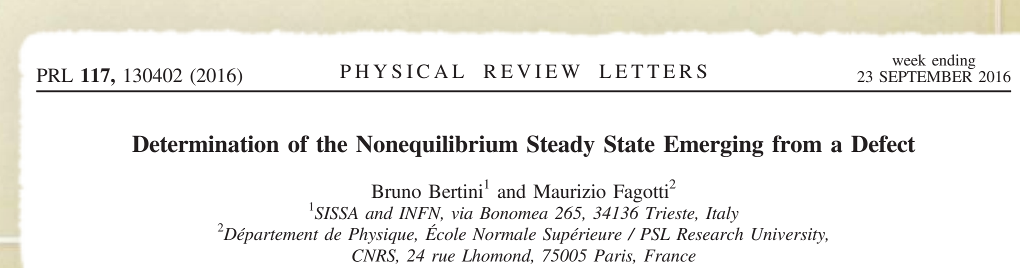
- The problem can be split in three steps:
  1. determine the macro-state emerging far away from the defect
  2. determine the NESS around the defect
  3. establish a connection between the stationary states emerging at different “rays”





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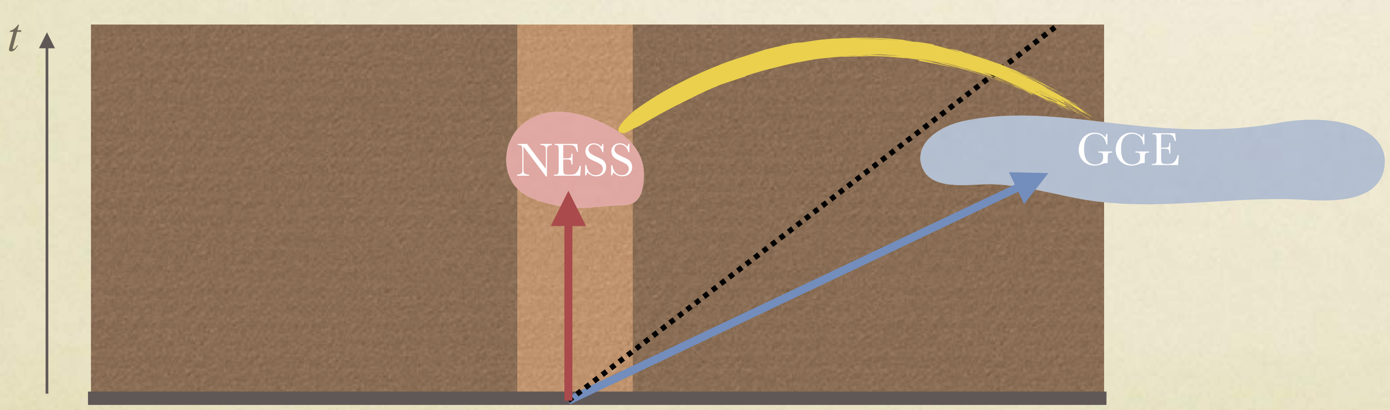

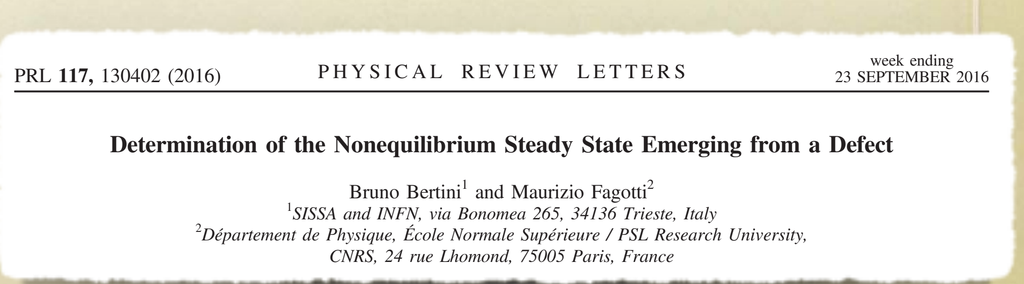
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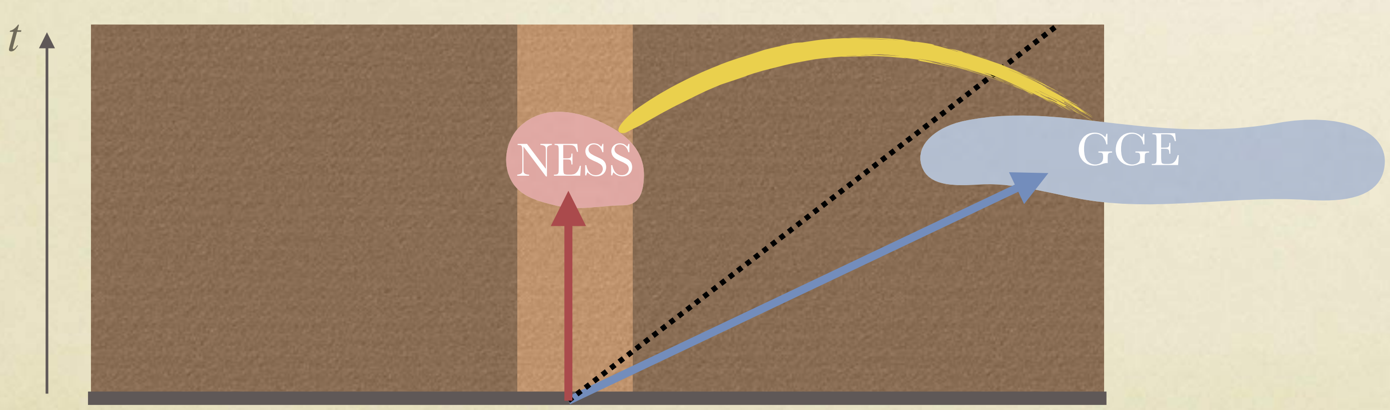

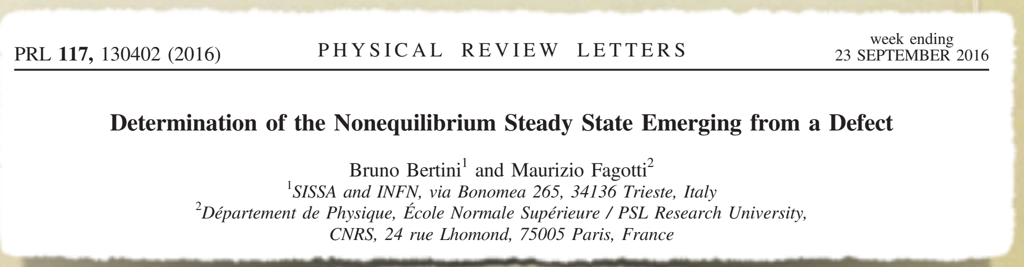
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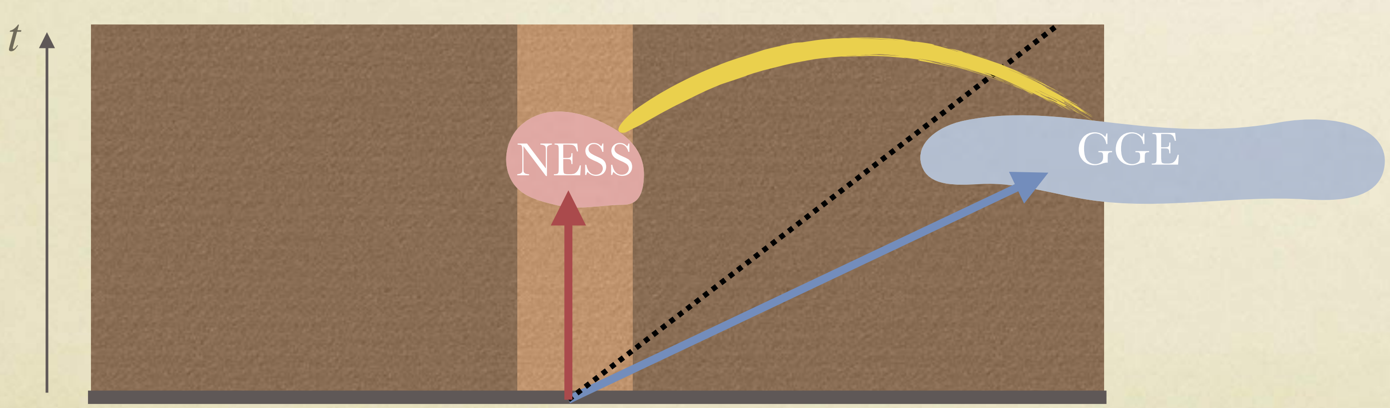

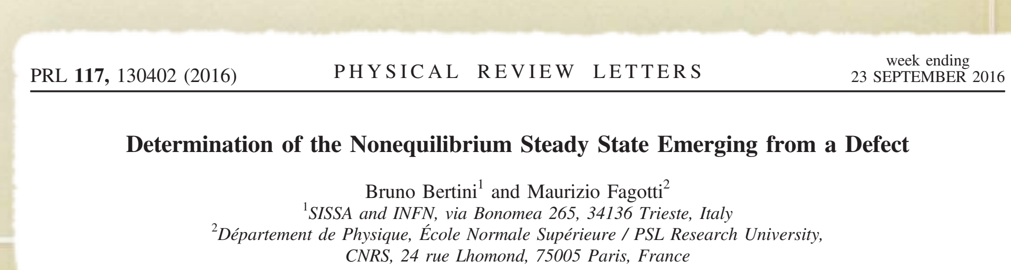

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locally quasistationary state

- homogeneous state
  homogeneous Hamiltonian with a localised defect

- domain-wall state
  homogeneous Hamiltonian

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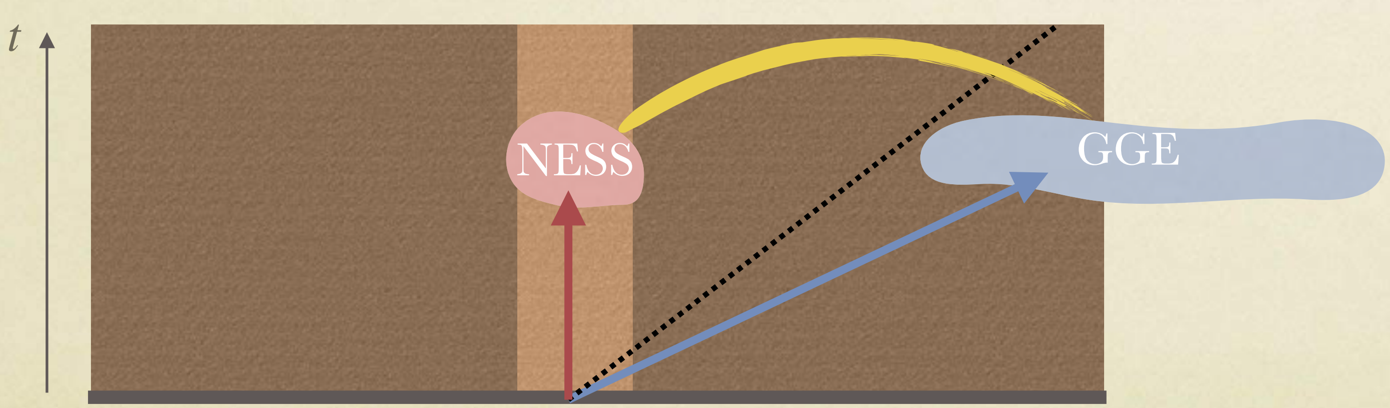

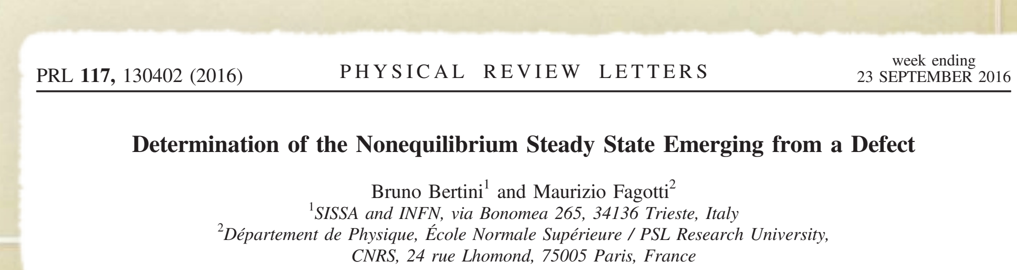
PRL 117, 130402 (2016) PHYSICAL REVIEW LETTERS week ending 23 SEPTEMBER 2016

Determination of the Nonequilibrium Steady State Emerging from a Defect

Bruno Bertini[1] and Maurizio Fagotti[2]
[1]*SISSA and INFN, via Bonomea 265, 34136 Trieste, Italy*
[2]*Département de Physique, École Normale Supérieure / PSL Research University, CNRS, 24 rue Lhomond, 75005 Paris, France*

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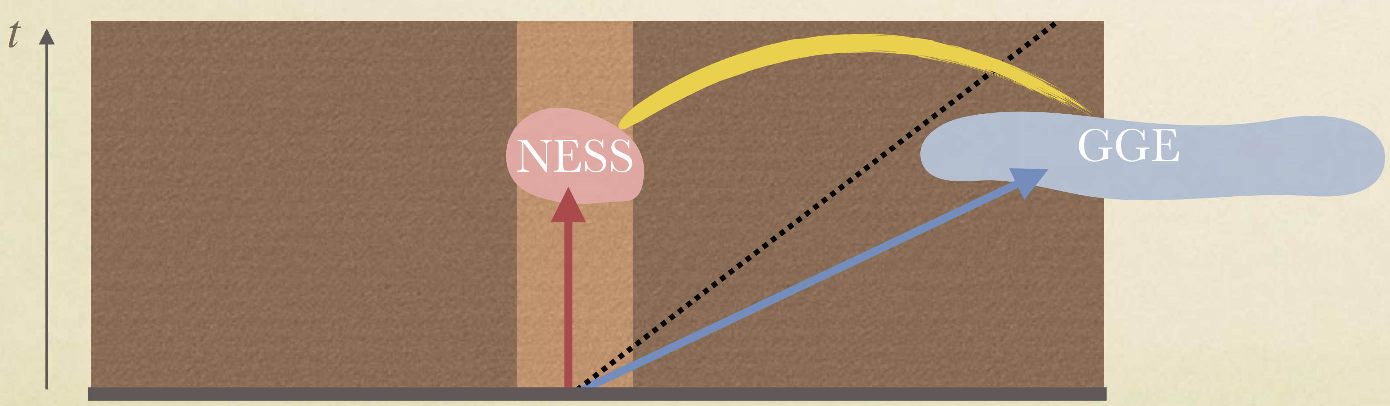

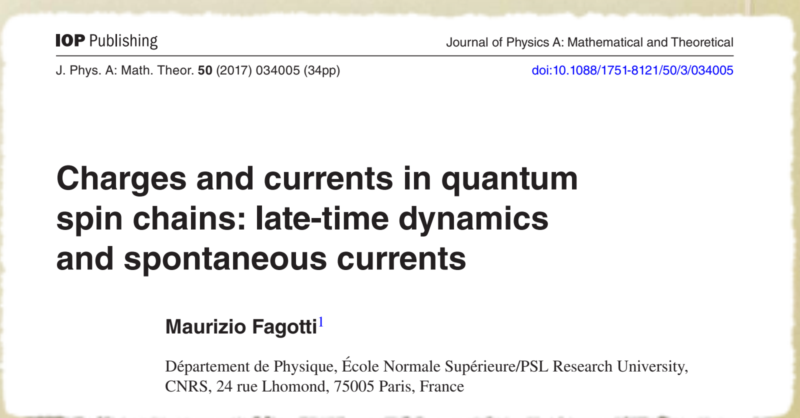
IOP Publishing Journal of Physics A: Mathematical and Theoretical
J. Phys. A: Math. Theor. 50 (2017) 034005 (34pp) doi:10.1088/1751-8121/50/3/034005

Charges and currents in quantum spin chains: late-time dynamics and spontaneous currents

Maurizio Fagotti[1]

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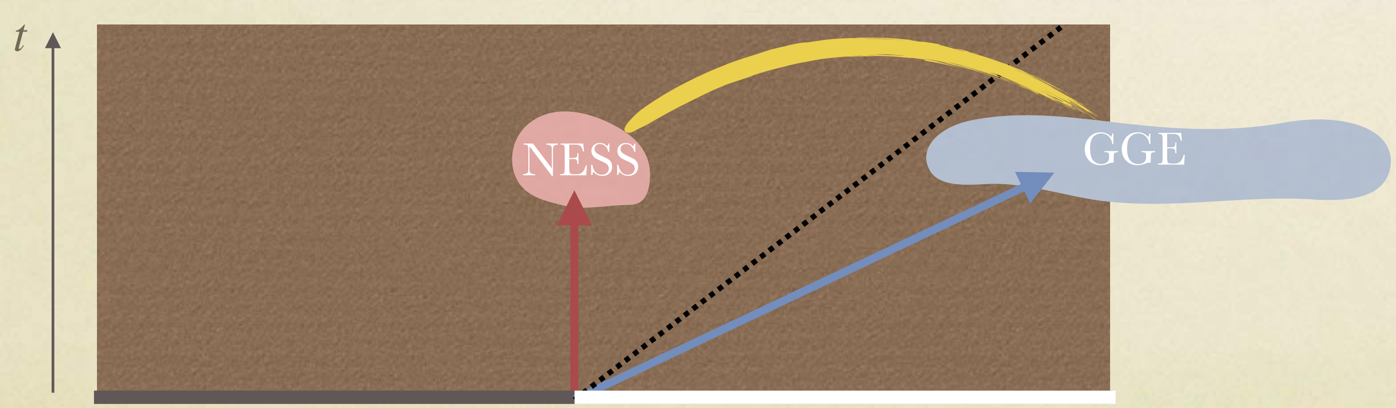

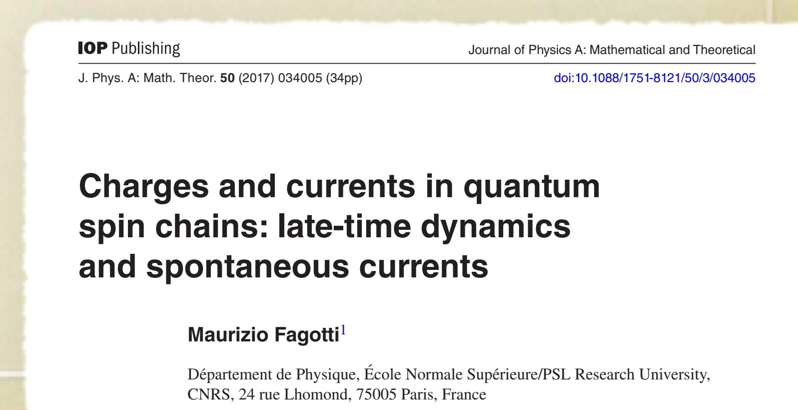

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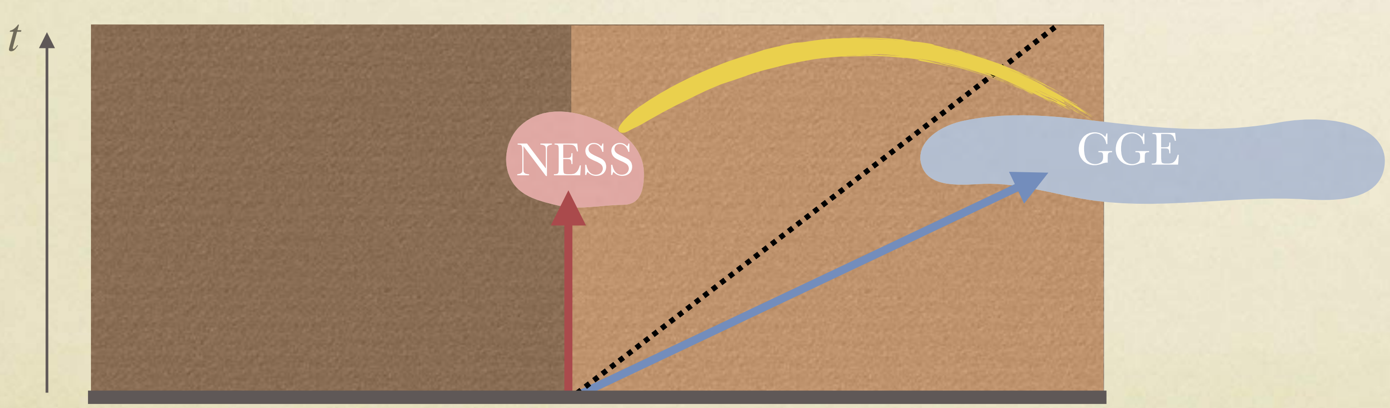

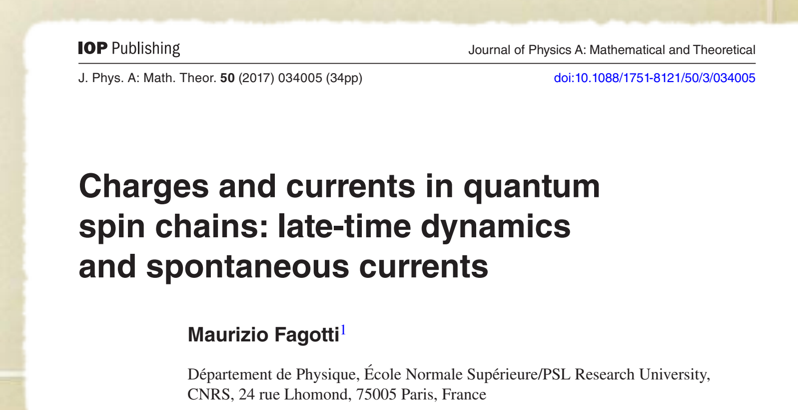

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CONTINUITY EQUATION

the expectation values of the charge densities completely characterise the space-time scaling limit

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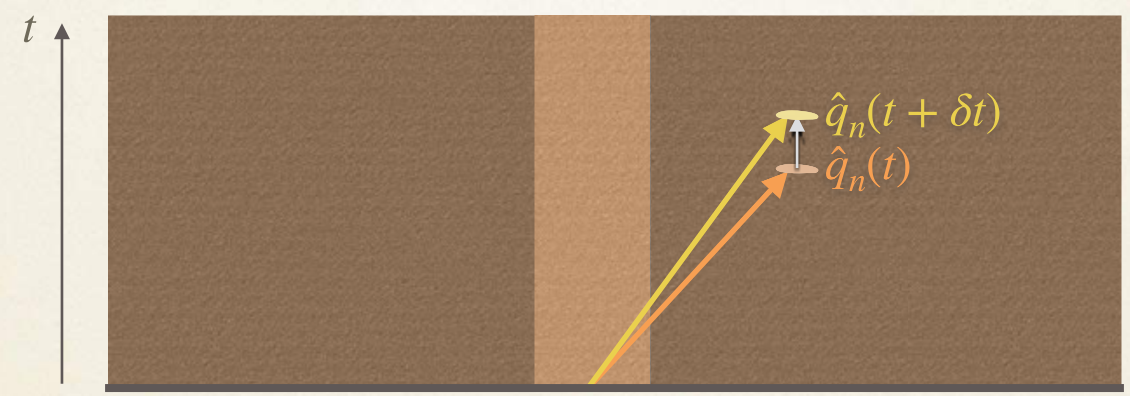

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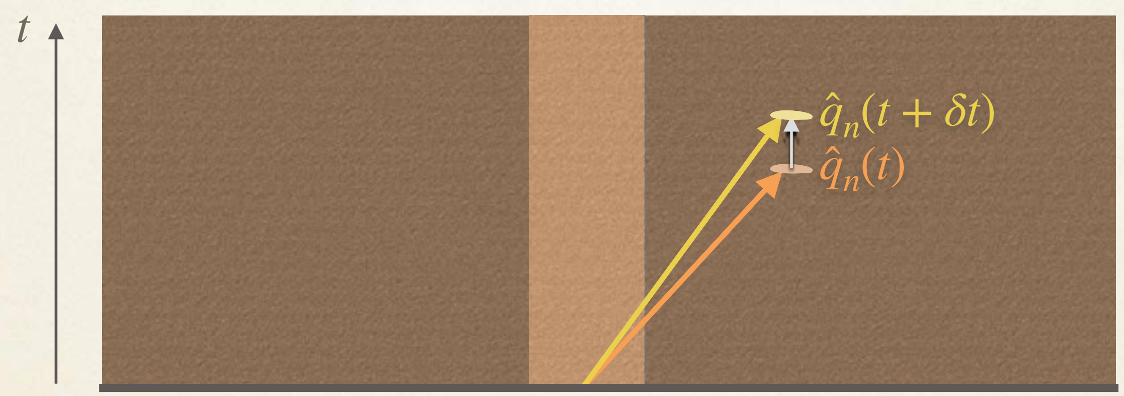

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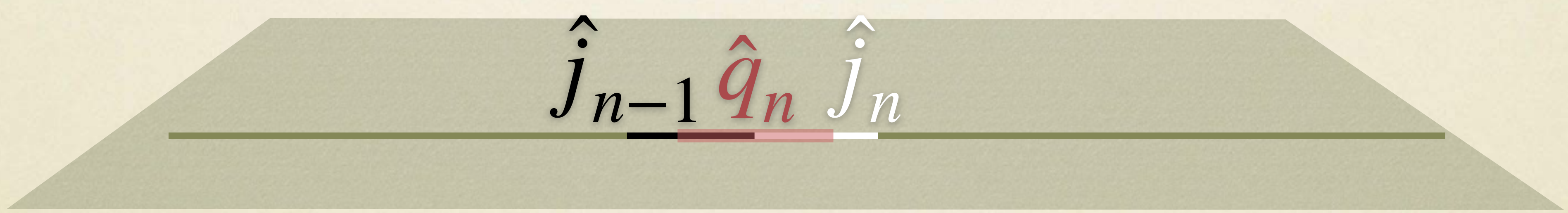

$$[\hat{H}, \hat{Q}] = 0 \qquad \hat{Q} = \sum_n \hat{q}_n$$

$$\partial_t \hat{q}_n + \hat{j}_n - \hat{j}_{n-1} = 0$$

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local relaxation:

$$\boxed{\left\langle \Psi_0 \right| \hat{q}_n(t) \left| \Psi_0 \right\rangle \to \mathrm{tr}[\hat{\rho}^{MS}_{n/t} \hat{q}_n] = \frac{1}{L} \mathrm{tr}[\hat{\rho}^{MS}_{n/t} \hat{Q}]}$$

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corrections from the extension of the range of charges and currents

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$$\begin{pmatrix} \partial_t f(x/t) = -\dfrac{1}{t}\dfrac{x}{t} f'(x/t) \\ \partial_x f(x/t) = \dfrac{1}{t} f'(x/t) \end{pmatrix}$$

LQSS:

$$\hat{J} \to \hat{J}^d \qquad \mathrm{tr}[(\zeta \hat{Q} - \hat{J}^d)\partial_\zeta \hat{\rho}^{MS}_\zeta] = o(t^{-1})$$

$$\zeta \partial_\zeta q[\{q^{(1)}_\xi, q^{(2)}_\xi, \dots\}] \approx \partial_\zeta j^d[\{q^{(1)}_\xi, q^{(2)}_\xi, \dots\}]$$

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REQUIREMENT

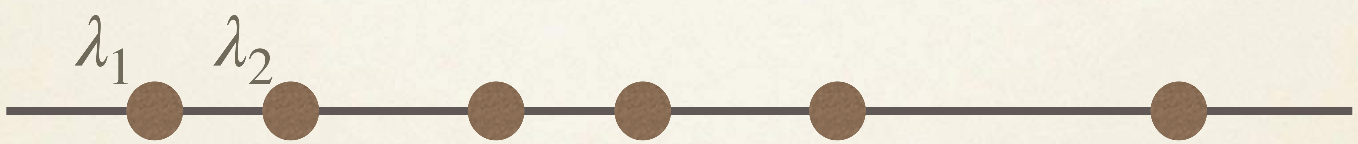
expectation values of charges and currents in a stationary state

Free fermion systems

$$\left|\lambda_1, \lambda_2, \ldots\right\rangle = [b^\dagger_{\lambda_1} b^\dagger_{\lambda_2} \cdots]\left|\varnothing\right\rangle$$

$$\{b^\dagger_\lambda, b_\mu\} = \delta_{\lambda\mu}$$
$$\{b^\dagger_\lambda, b^\dagger_\mu\} = 0$$

energy	momentum	local charge*
$E = \sum_n e(\lambda_n)$	$P = \sum_n p(\lambda_n)$	$Q = \sum_n q(\lambda_n)$



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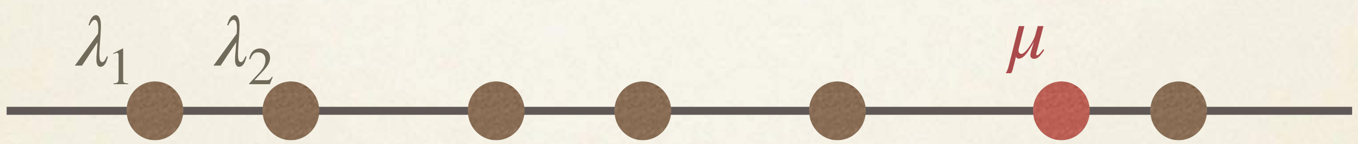

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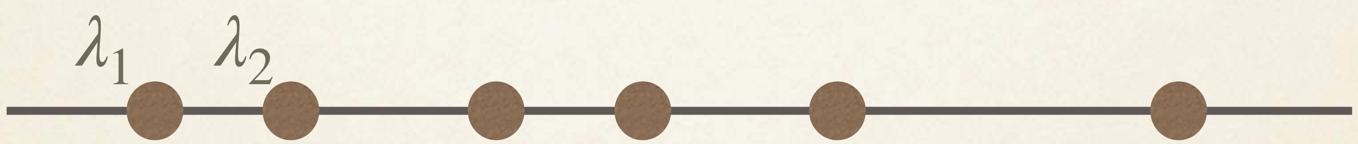

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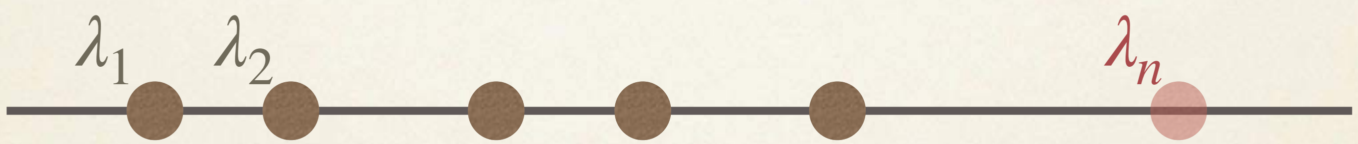

<u>Interacting</u> integrable systems

$$\left|\lambda_1, \lambda_2, \ldots\right\rangle = [B(\lambda_1)B(\lambda_2)\cdots]\left|\varnothing\right\rangle$$

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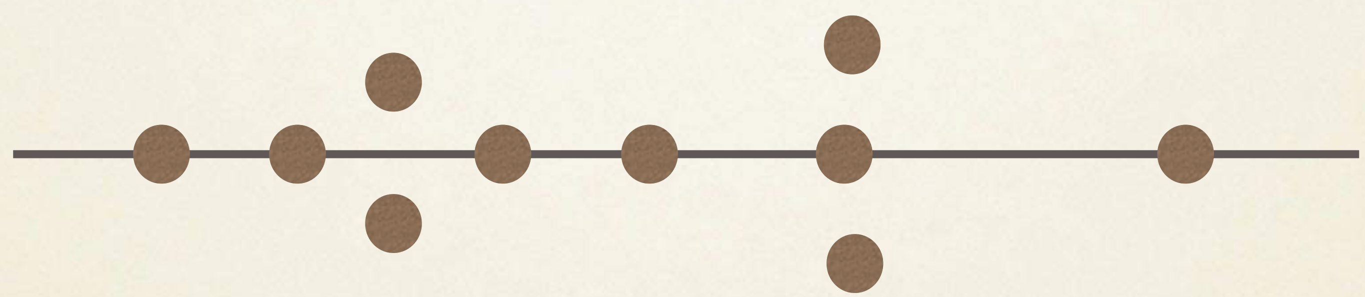

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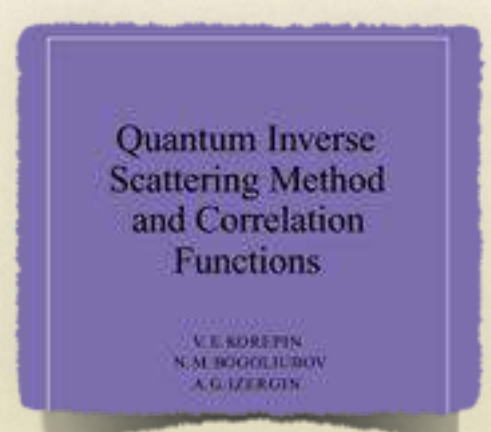

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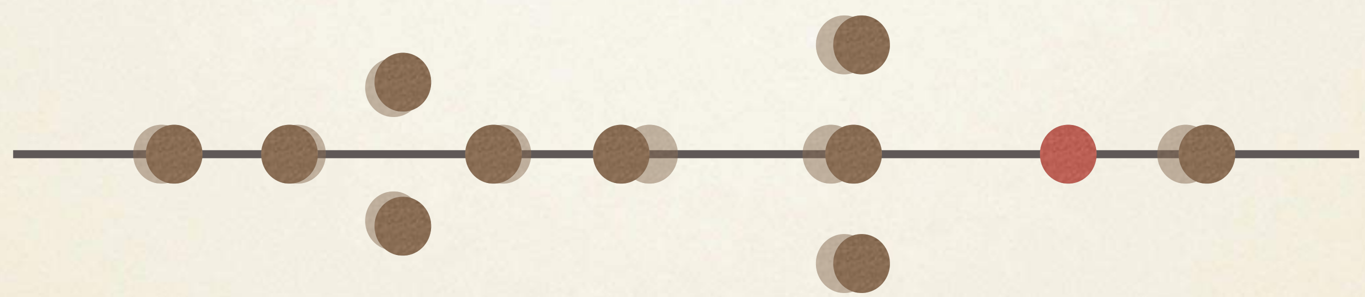

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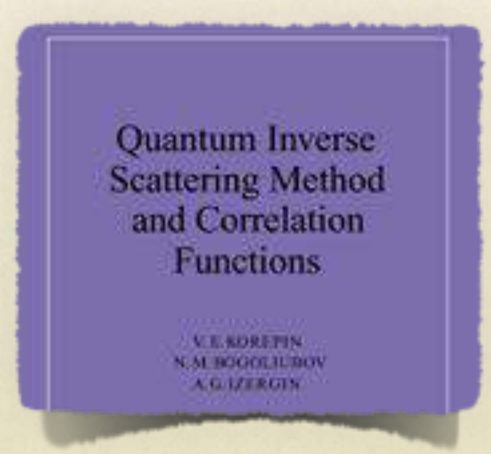

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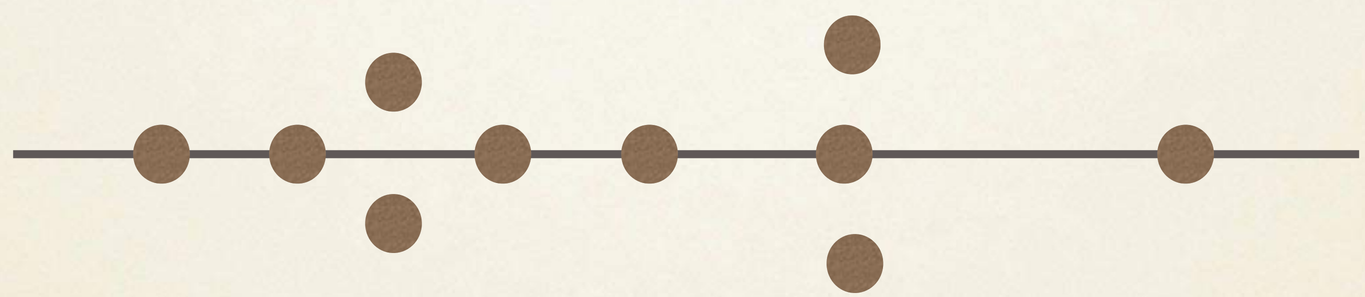

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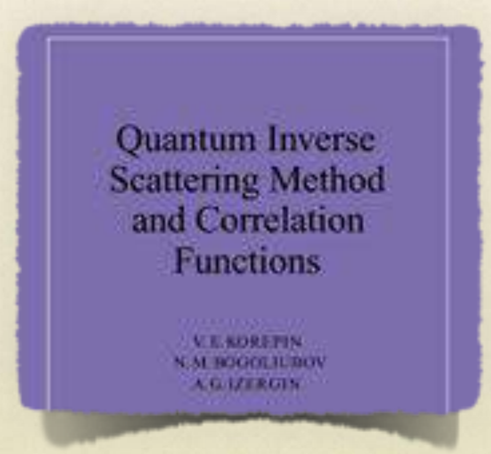

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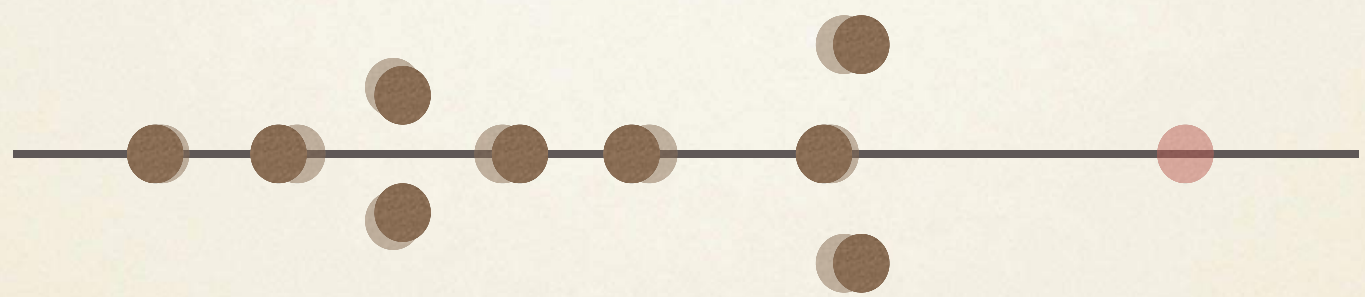

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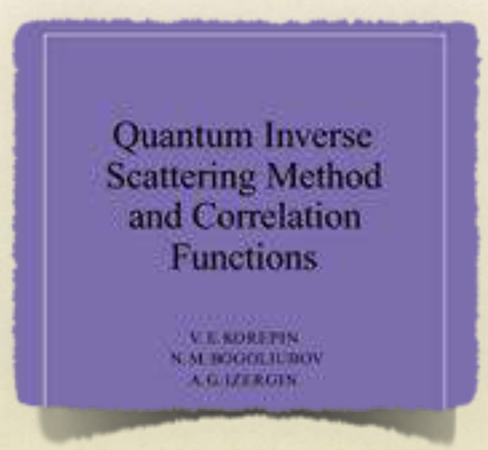

Time Evolution and Excitations

$$\left\langle \Psi_0 \right| e^{i\hat{H}T} \hat{q}_X e^{-i\hat{H}T} \left| \Psi_0 \right\rangle = \sum_{\{\lambda\},\{\tilde{\lambda}\}} \left\langle \Psi_t \middle| \{\lambda\} \right\rangle \left\langle \{\tilde{\lambda}\} \middle| \Psi_t \right\rangle \left\langle \{\lambda\} \right| \hat{q}_x \left| \{\tilde{\lambda}\} \right\rangle e^{i(E_{\{\lambda\}}-E_{\{\tilde{\lambda}\}})(T-t)} e^{-i(P_{\{\lambda\}}-P_{\{\tilde{\lambda}\}})(X-x)} =$$

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local observable

$$\{\lambda\} \sim \{\tilde{\lambda}\}$$

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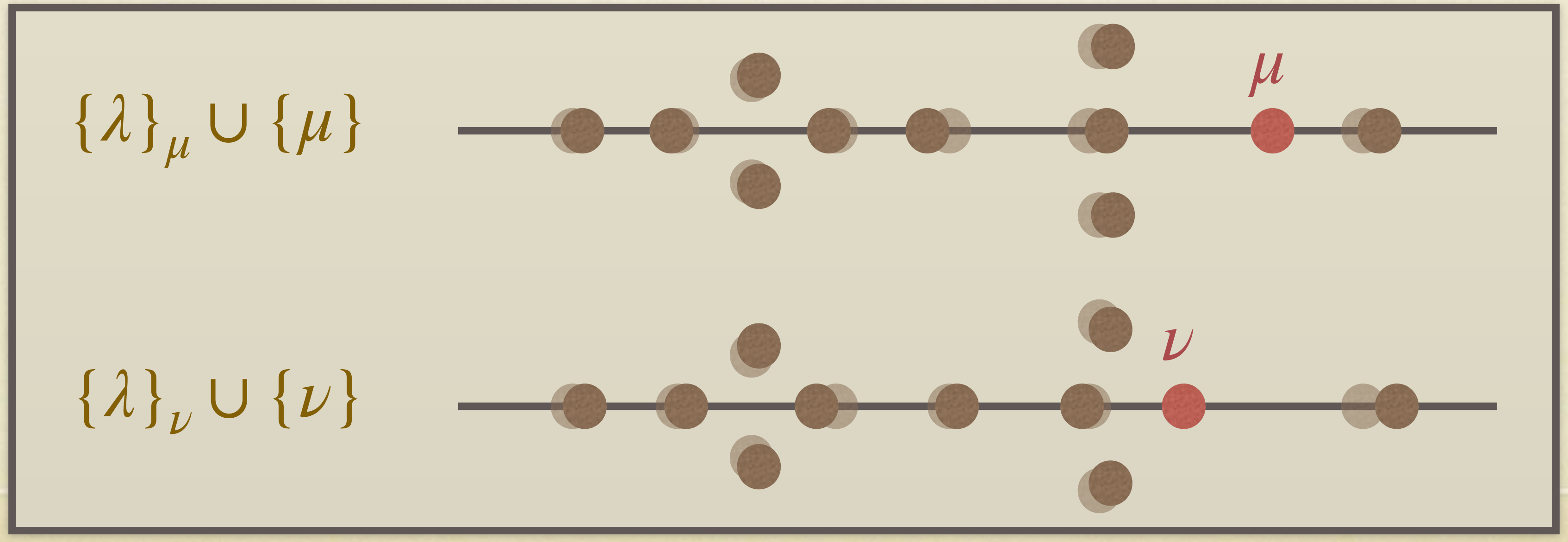
$$\varepsilon_{\{\lambda\}}(\mu) = E_{\{\lambda\}_\mu \cup \{\mu\}} - E_{\{\lambda\}}$$

$$\pi_{\{\lambda\}}(\mu) = P_{\{\lambda\}_\mu \cup \{\mu\}} - P_{\{\lambda\}}$$

$$= \sum_{n,m} \int \mathrm{d}^n\mu \int \mathrm{d}^m\nu \sum_{\{\lambda\}} \left\langle \Psi_t \middle| \{\lambda\}_\mu \cup \{\mu\} \right\rangle \left\langle \{\lambda\}_\nu \cup \{\nu\} \middle| \Psi_t \right\rangle \left\langle \{\lambda\}_\mu \cup \{\mu\} \right| \hat{q}_x \left| \{\lambda\}_\nu \cup \{\nu\} \right\rangle e^{i\sum_j [\varepsilon_{\{\lambda\}}(\mu_j)(T-t) - \pi_{\{\lambda\}}(\mu_j)(X-x)] - i\sum_j [\varepsilon_{\{\lambda\}}(\nu_j)(T-t) - \pi_{\{\lambda\}}(\nu_j)(X-x)]}$$



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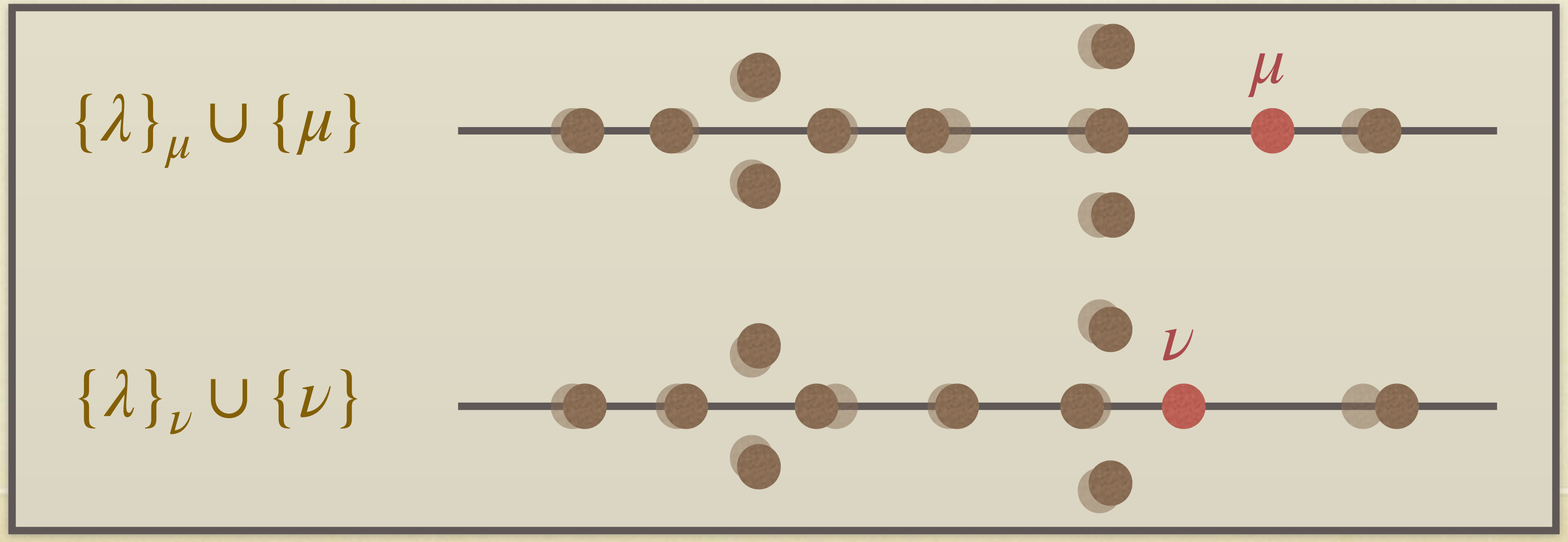
$$\left| \Psi_t \right\rangle \to \hat{\rho}^{MS}_{x/t}$$



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$$\left| \Psi_t \right\rangle \to \hat{\rho}^{MS}_{x/t}$$

$$v_{\{\lambda\}}(\mu_j) = \frac{\varepsilon'_{\{\lambda\}}(\mu_j)}{\pi'_{\{\lambda\}}(\mu_j)} = \frac{X-x}{T-t}$$

it's bounded!
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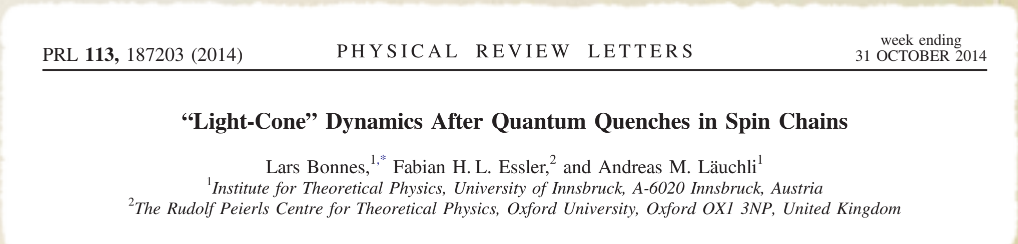
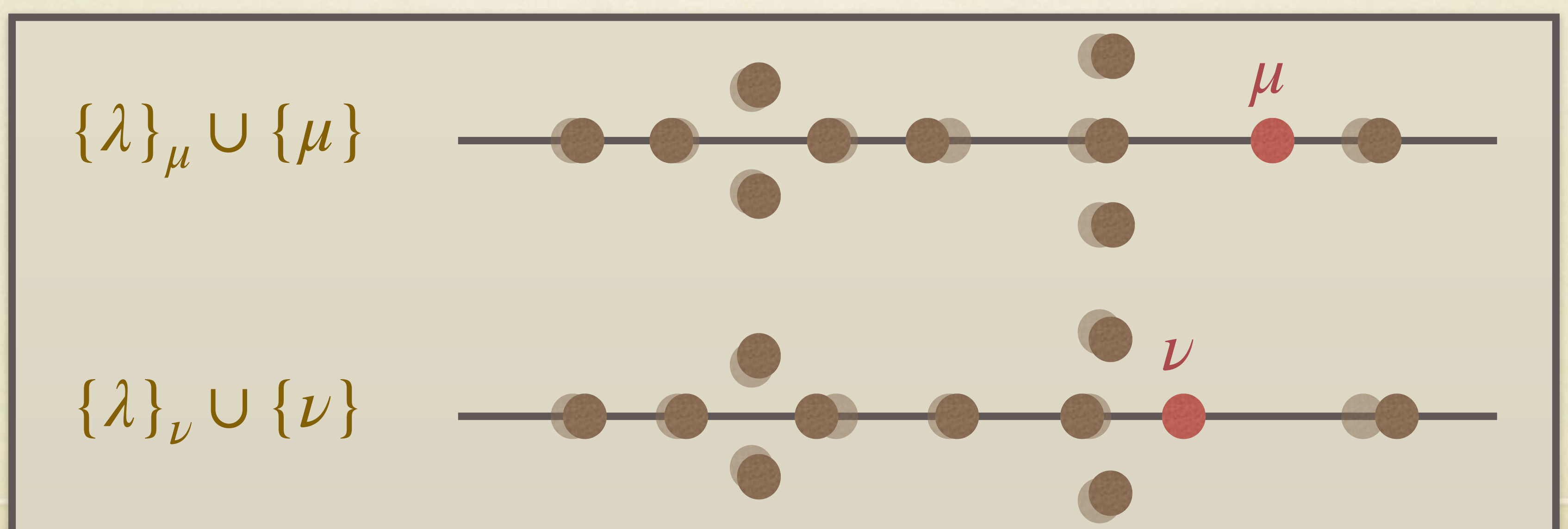

PRL 113, 187203 (2014) PHYSICAL REVIEW LETTERS week ending 31 OCTOBER 2014

"Light-Cone" Dynamics After Quantum Quenches in Spin Chains

Lars Bonnes,[1,*] Fabian H. L. Essler,[2] and Andreas M. Läuchli[1]
[1]*Institute for Theoretical Physics, University of Innsbruck, A-6020 Innsbruck, Austria*
[2]*The Rudolf Peierls Centre for Theoretical Physics, Oxford University, Oxford OX1 3NP, United Kingdom*



Time Evolution and Excitations

$$\left\langle \Psi_0 \right| e^{i\hat{H}T} \hat{q}_X e^{-i\hat{H}T} \left| \Psi_0 \right\rangle = \sum_{\{\lambda\},\{\tilde{\lambda}\}} \left\langle \Psi_t \middle| \{\lambda\} \right\rangle \left\langle \{\tilde{\lambda}\} \middle| \Psi_t \right\rangle \left\langle \{\lambda\} \right| \hat{q}_x \left| \{\tilde{\lambda}\} \right\rangle e^{i(E_{\{\lambda\}} - E_{\{\tilde{\lambda}\}})(T-t)} e^{-i(P_{\{\lambda\}} - P_{\{\tilde{\lambda}\}})(X-x)} =$$

local observable

$\{\lambda\} \sim \{\tilde{\lambda}\}$

$$\varepsilon_{\{\lambda\}}(\mu) = E_{\{\lambda\}_\mu \cup \{\mu\}} - E_{\{\lambda\}}$$
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ray relaxation

$\left| \Psi_t \right\rangle \to \hat{\rho}^{MS}_{x/t}$

velocity fixed

$$v_{\{\lambda\}}(\mu_j) = \frac{\varepsilon'_{\{\lambda\}}(\mu_j)}{\pi'_{\{\lambda\}}(\mu_j)} = \frac{X-x}{T-t}$$

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$t \sim T \Rightarrow \dfrac{x}{t}$ fixed

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$\mathbf{A}$: $\mathrm{tr}[\rho^{MS}\hat{q}_x] \rightarrow \rho^{MS}$

$\mathbf{B}$: $\rho^{MS} \rightarrow v^{MS}$

interacting integrable

noninteracting

Currents in Free Fermion Systems

example: $\hat{H} = \sum_{\ell=1}^{L} \frac{1}{2}(\hat{c}_\ell \hat{c}^\dagger_{\ell+1} - \hat{c}^\dagger_\ell \hat{c}_{\ell+1}) + \frac{\gamma}{2}(\hat{c}_\ell \hat{c}_{\ell+1} - \hat{c}^\dagger_\ell \hat{c}^\dagger_{\ell+1})$

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$$a_{2\ell-1} = c_\ell + c_\ell^\dagger$$
$$a_{2\ell} = i(c_\ell - c_\ell^\dagger)$$

block-Toeplitz operator *(with $\ell, n \in \mathbb{Z}$)* *symbol*

$$\mathbb{T}[\mathbf{t}]_{2\kappa\ell+i, 2\kappa n+j} = \int \frac{\mathrm{d}\lambda}{2\pi} e^{-i(\ell-n)\lambda} [\mathbf{t}(\lambda)]_{ij}$$

Hamiltonian $\hat{H} = \frac{1}{4} \vec{a}^{\,t} \mathbb{T}[\mathbf{h}] \vec{a}$

local charges $\hat{Q} = \frac{1}{4} \vec{a}^{\,t} \mathbb{T}[\mathbf{q}] \vec{a}$ $[\mathbf{h}(\lambda), \mathbf{q}(\lambda)] = 0$


IOP Publishing Journal of Physics A: Mathematical and Theoretical
J. Phys. A: Math. Theor. **50** (2017) 034005 (34pp) doi:10.1088/1751-8121/50/3/034005

Charges and currents in quantum spin chains: late-time dynamics and spontaneous currents

Maurizio Fagotti[1]

Département de Physique, École Normale Supérieure/PSL Research University, CNRS, 24 rue Lhomond, 75005 Paris, France

Currents in Free Fermion Systems

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dispersion relation of the charge

$$\frac{1}{L}\langle\{\lambda\}|\,\hat{Q}\,|\{\lambda\}\rangle = \frac{1}{L}\sum_j q(\lambda_j) \to \int \mathrm{d}\lambda \rho(\lambda) q(\lambda)$$

density of rapidities occupied

$$\frac{1}{L}\langle\{\lambda\}|\,\hat{J}\,|\{\lambda\}\rangle = \frac{1}{L}\sum_j v(\lambda_j) q(\lambda_j) \to \int \mathrm{d}\lambda \rho(\lambda) v(\lambda) q(\lambda)$$

velocity

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Currents in Free Fermion Systems
quasiparticle interpretation

- macro-state represented by the density of quasiparticle excitations $\rho(\lambda)$

- the density $\rho(\lambda)$ is completely characterised by the local integrals of motion
  [the set $\{q(\lambda)\}$ *forms a complete basis]*

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the density of quasiparticles in the rapidity space is conserved

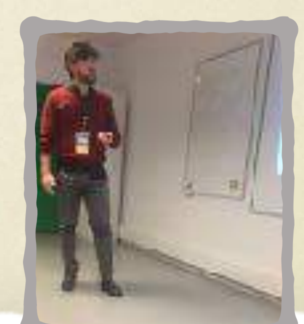
scattering is elastic!

GHD in Interacting Integrable Systems

$$\zeta\partial_\zeta \rho_{n,\zeta}(\lambda) \approx \partial_\zeta[v_{n,\zeta}(\lambda)\rho_{n,\zeta}(\lambda)]$$

generally there are more species of excitations

the velocity depends on the ray, as it depends on the state
[the relation is provided by the Bethe Ansatz]



PHYSICAL REVIEW X **6,** 041065 (2016)

Emergent Hydrodynamics in Integrable Quantum Systems Out of Equilibrium

Olalla A. Castro-Alvaredo,[1] Benjamin Doyon,[2] and Takato Yoshimura[2]
[1]*Department of Mathematics, City, University of London, Northampton Square, London EC1V 0HB, United Kingdom*
[2]*Department of Mathematics, King's College London, Strand, London WC2R 2LS, United Kingdom*

PRL **117,** 207201 (2016) PHYSICAL REVIEW LETTERS week ending 11 NOVEMBER 2016

Transport in Out-of-Equilibrium *XXZ* Chains: Exact Profiles of Charges and Currents

Bruno Bertini,[1] Mario Collura,[1,2] Jacopo De Nardis,[3] and Maurizio Fagotti[3]
[1]*SISSA and INFN, via Bonomea 265, 34136 Trieste, Italy*
[2]*The Rudolf Peierls Centre for Theoretical Physics, Oxford University, Oxford, OX1 3NP, United Kingdom*
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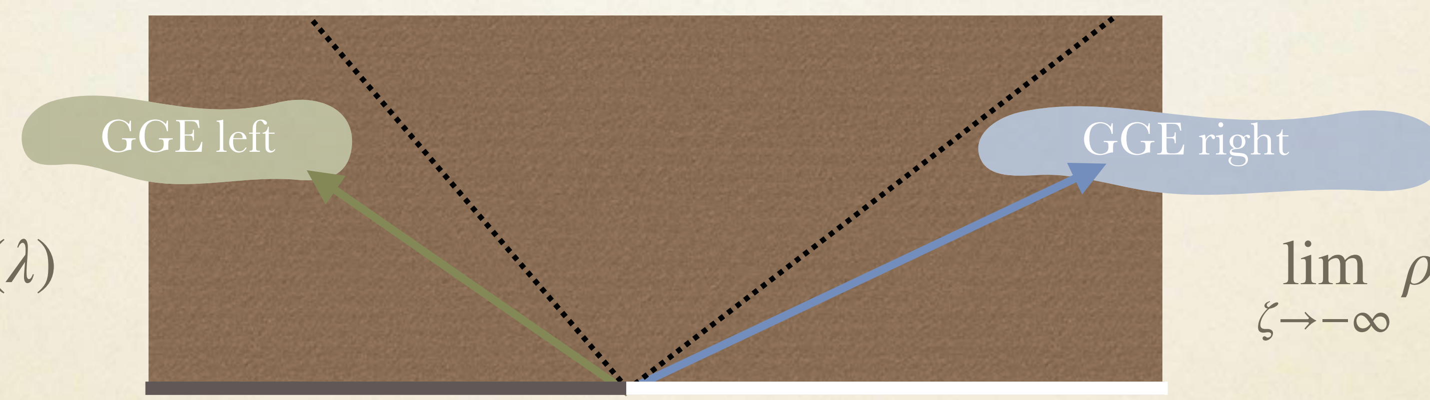
boundary conditions

$$\lim_{\zeta \to -\infty} \rho_{n,\zeta}(\lambda) = \rho_n^{MS,\text{ left}}(\lambda)$$



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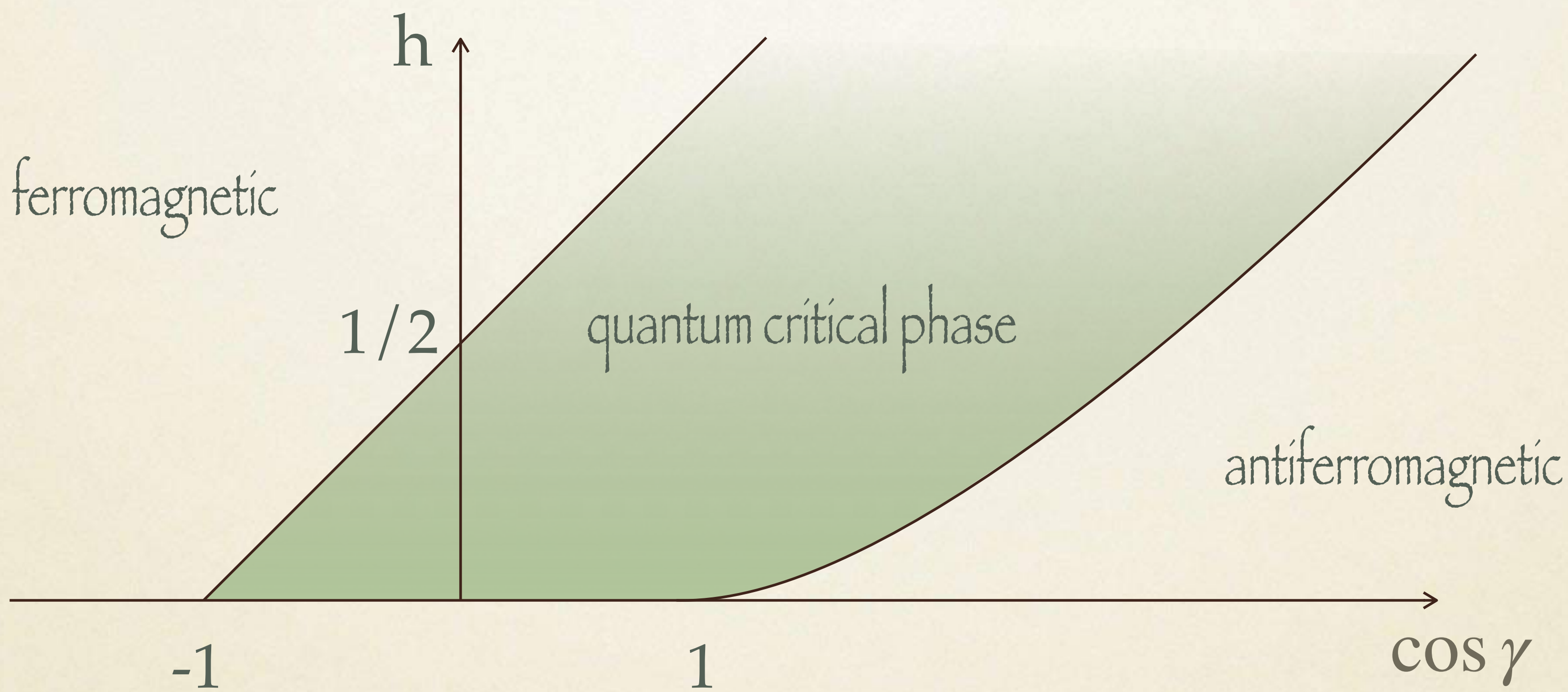
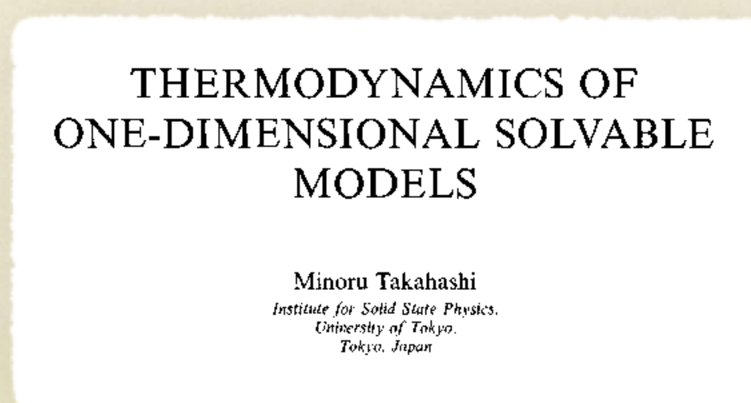
Example

anisotropy

XXZ spin-$\frac{1}{2}$ chain $$H = \sum_{\ell} \vec{s}_\ell \cdot \vec{s}_{\ell+1} - 2\sin^2\frac{\gamma}{2} s^z_\ell s^z_{\ell+1}$$




THERMODYNAMICS OF
ONE-DIMENSIONAL SOLVABLE
MODELS

Minoru Takahashi
Institute for Solid State Physics,
University of Tokyo,
Tokyo, Japan

Example
thermal transport

anisotropy

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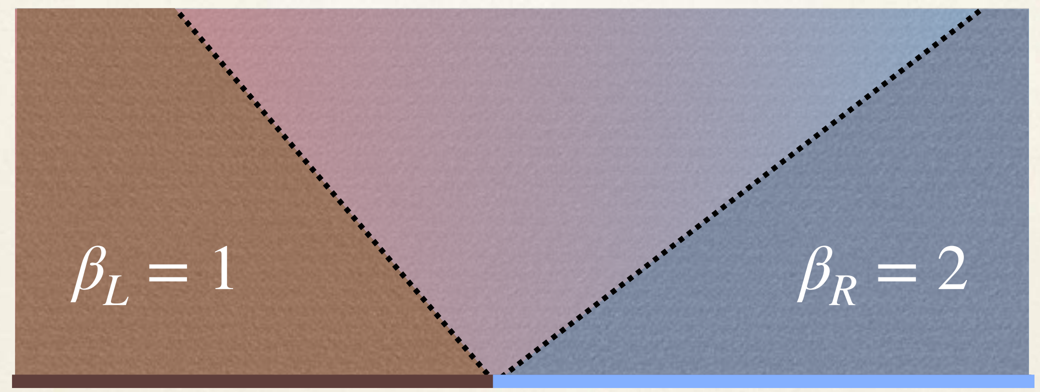

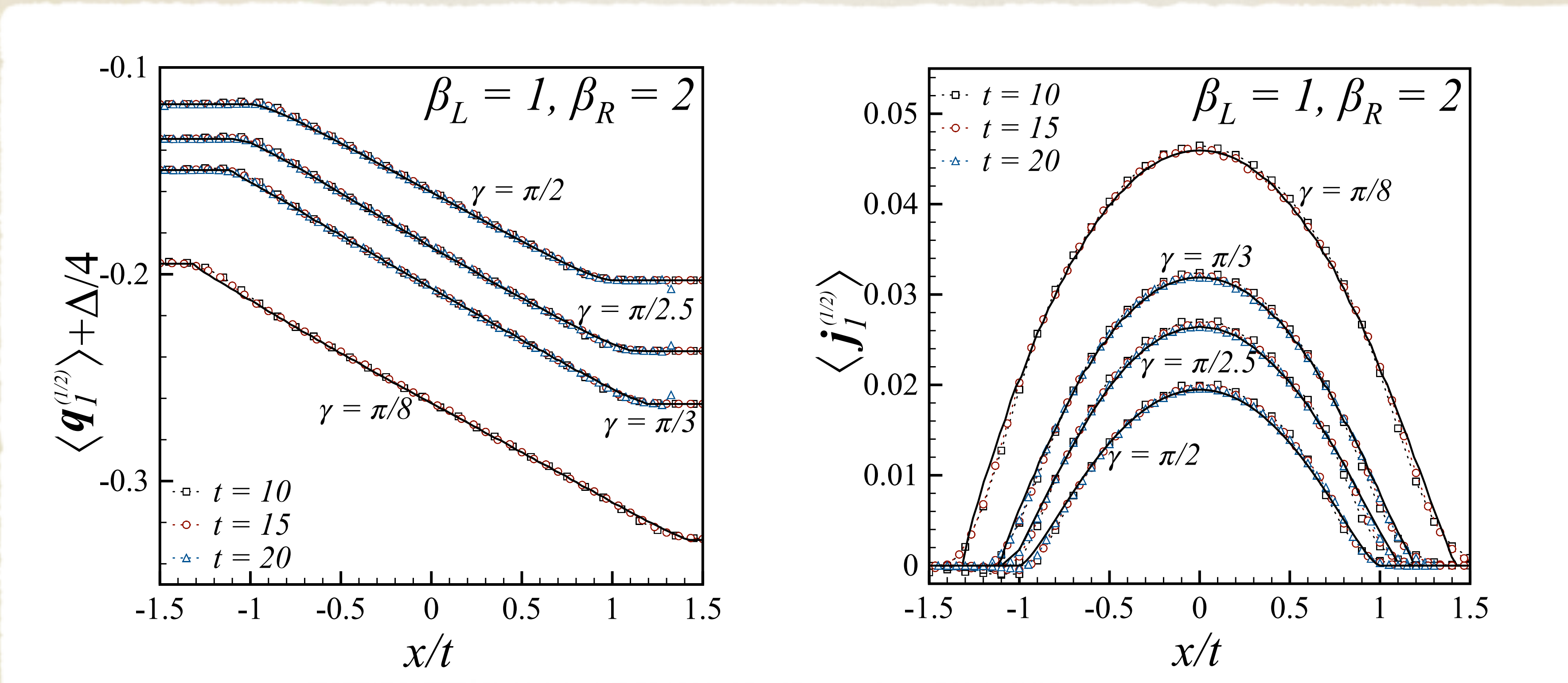

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spin transport

anisotropy

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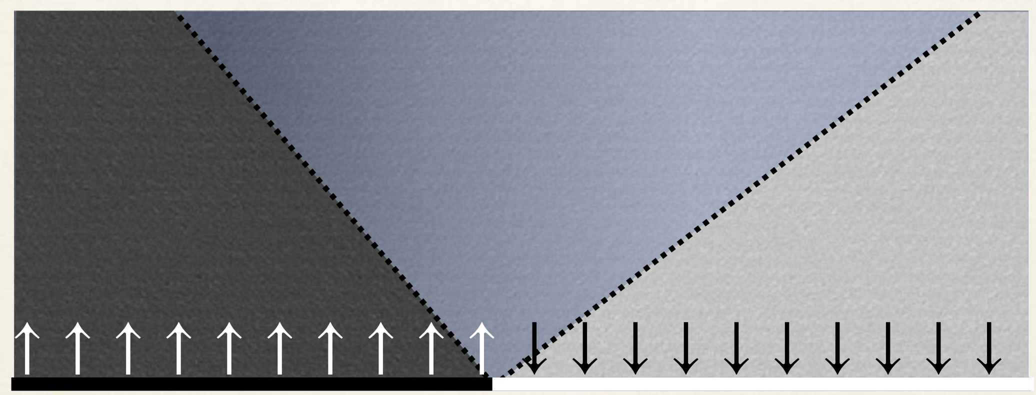

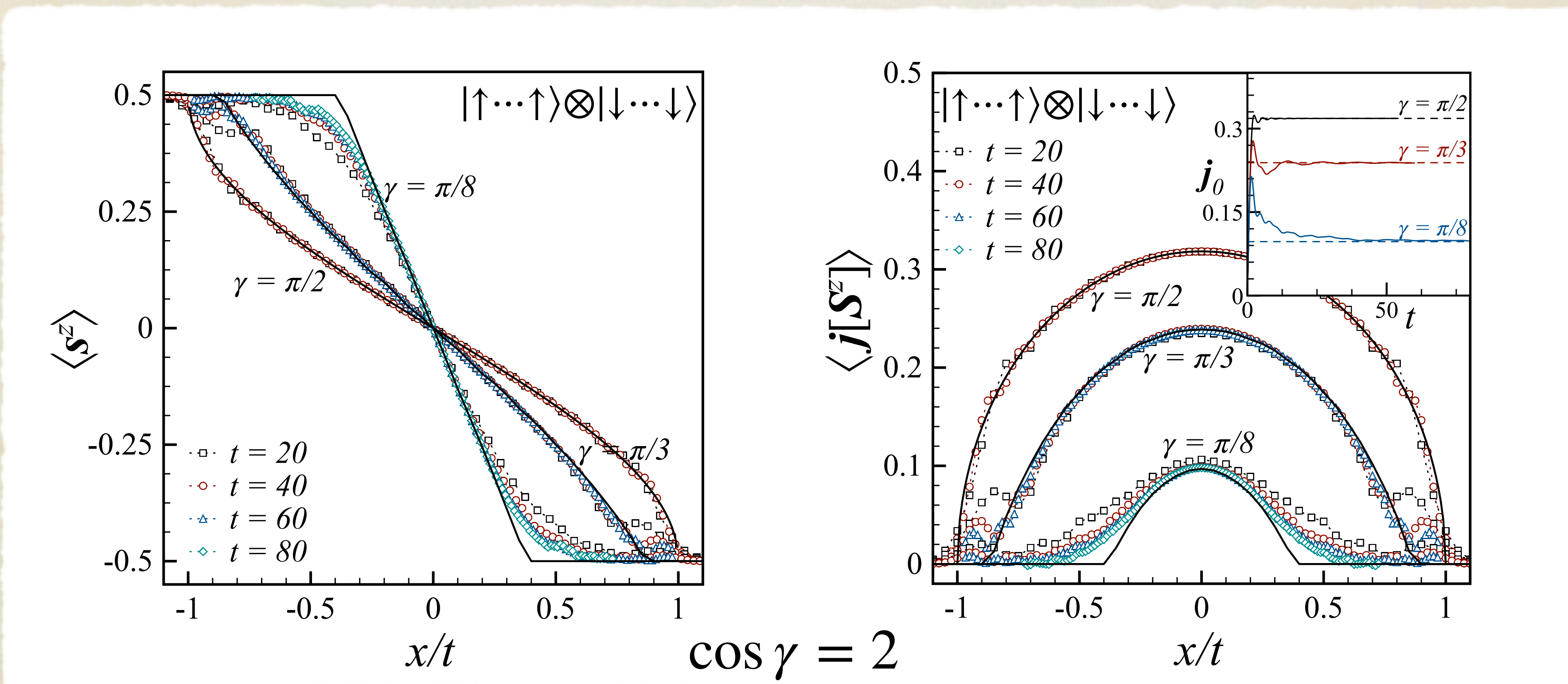

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beyond GHD

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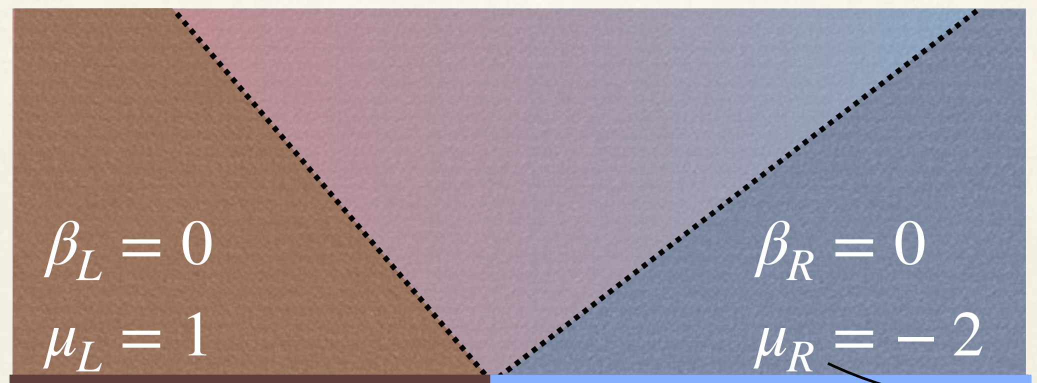
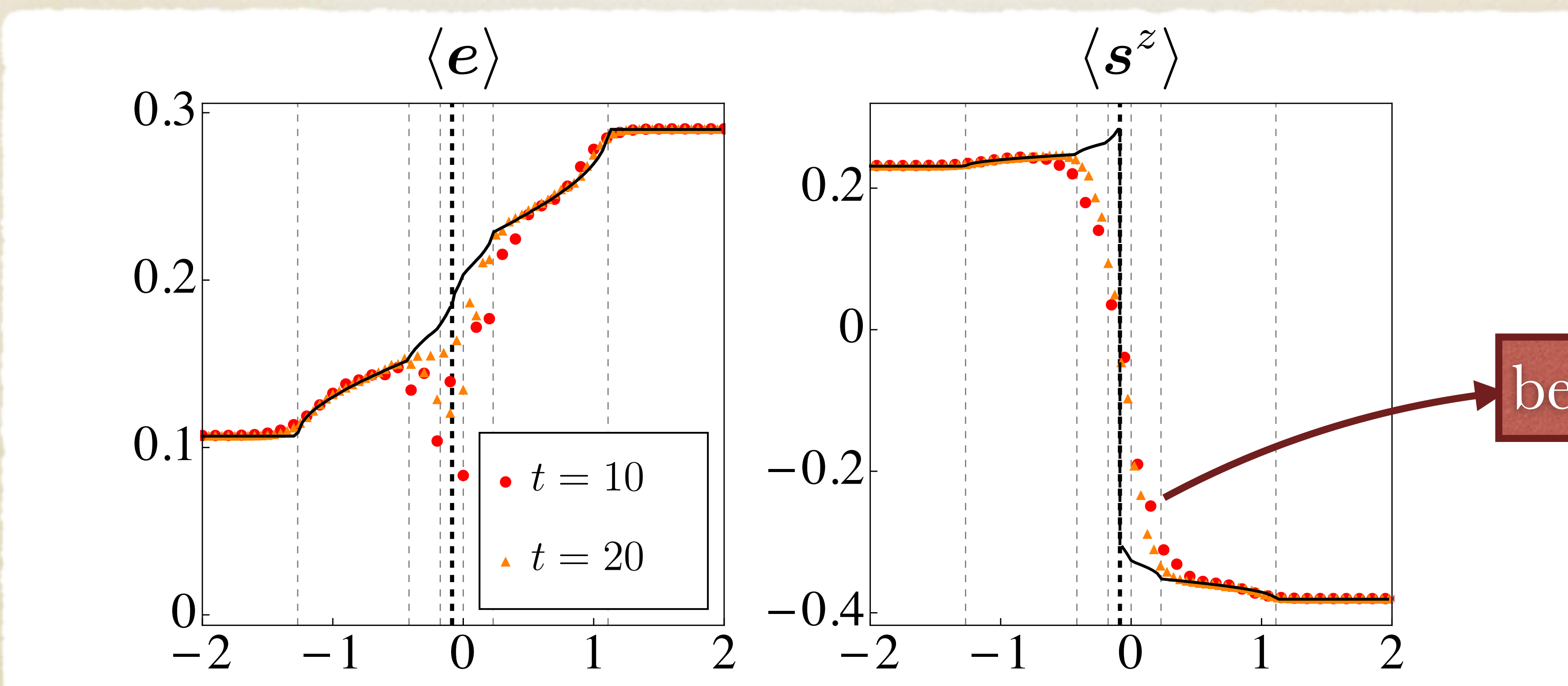


chemical potential



beyond GHD

PHYSICAL REVIEW B **96**, 115124 (2017)

Transport in out-of-equilibrium XXZ chains: Nonballistic behavior and correlation functions

Lorenzo Piroli,[1] Jacopo De Nardis,[2] Mario Collura,[3] Bruno Bertini,[1] and Maurizio Fagotti[2]

Recap

- A local defect can have macroscopic effects on the late-time dynamics

- The space-time scaling limit ($t \to \infty$, $\zeta = x/t$ fixed) in integrable systems is captured by a (generalised) hydrodynamic theory

- GHD does **not** capture diffusion and large-time corrections

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