THOULESS AND RELAXATION TIME SCALES IN MANY-BODY QUANTUM SYSTEMS

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OUTLINE

- Introduction
- Time scales for relaxation in GOE random matrices
- Evolution of the survival probability for spin chain
- Evolution of the spin autocorrelation function
- Conclusion

INTRODUCTION

How does a generic chaotic system relax to its asymptotic steady state?

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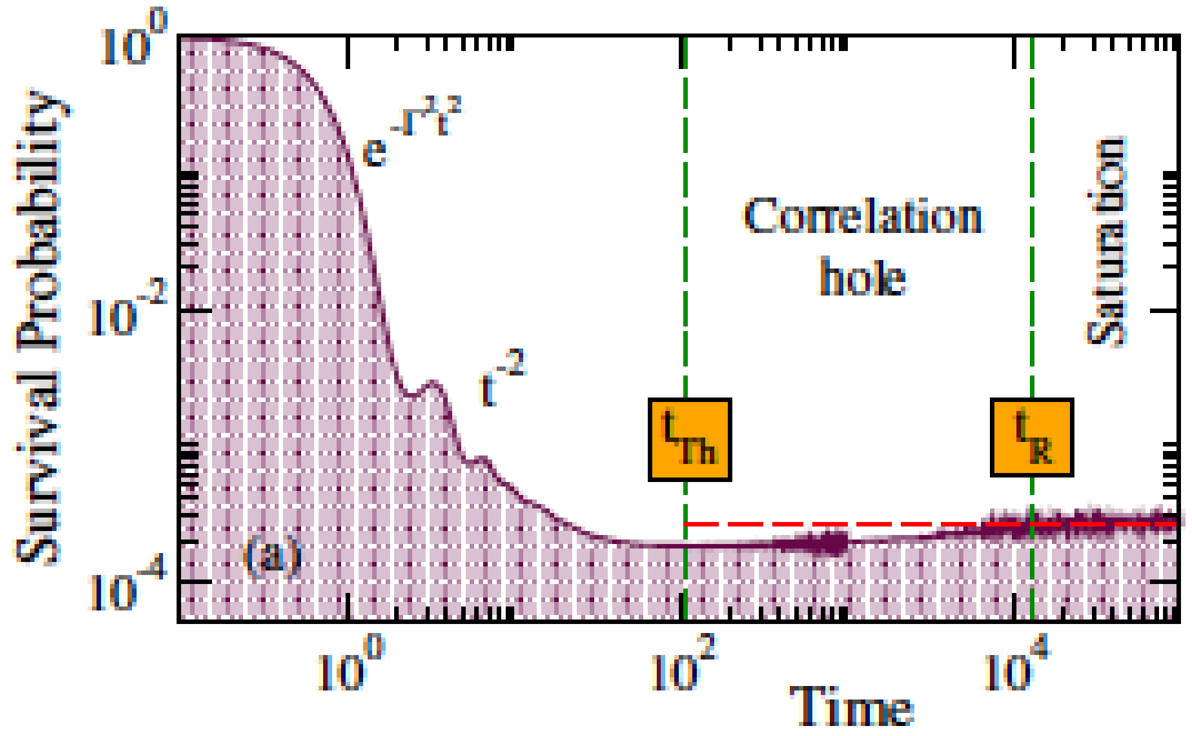

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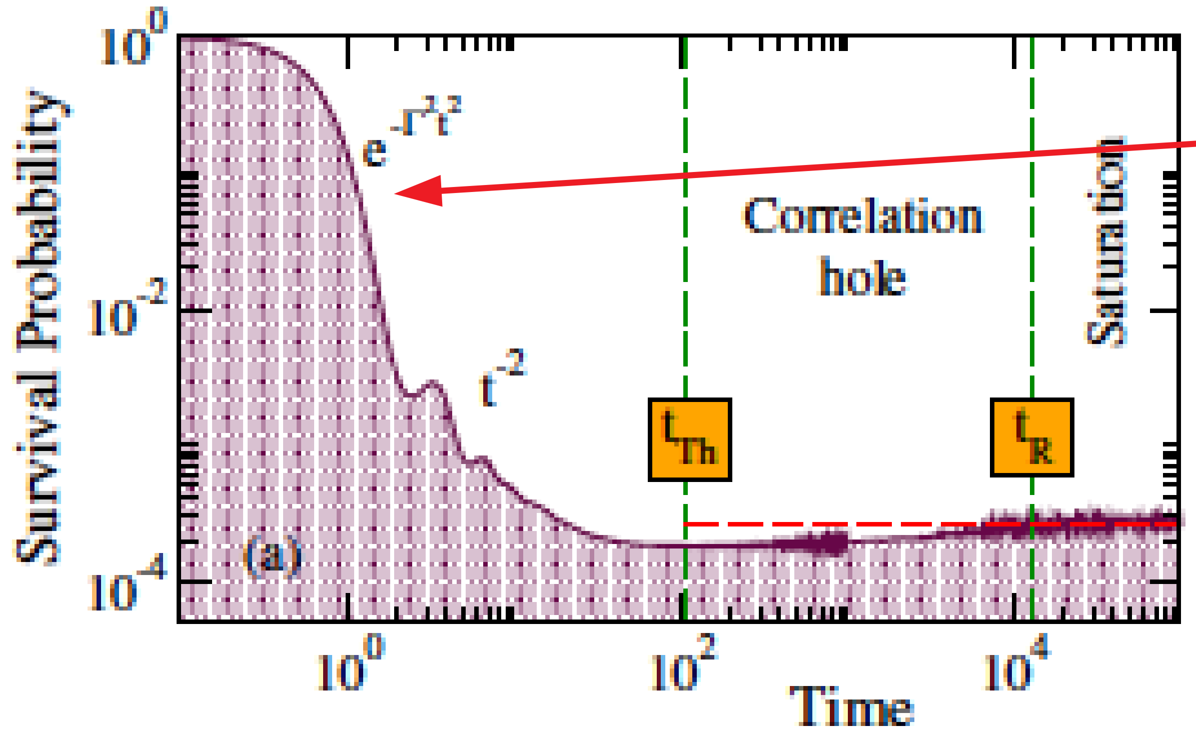

Step one: fast depletion of the initial state

Time scale: inverse of the width Γ of the Local Density of States (LDOS)

$$\rho_0(E) = \sum_\alpha |c_\alpha^{(0)}| \delta(E - E_\alpha)$$

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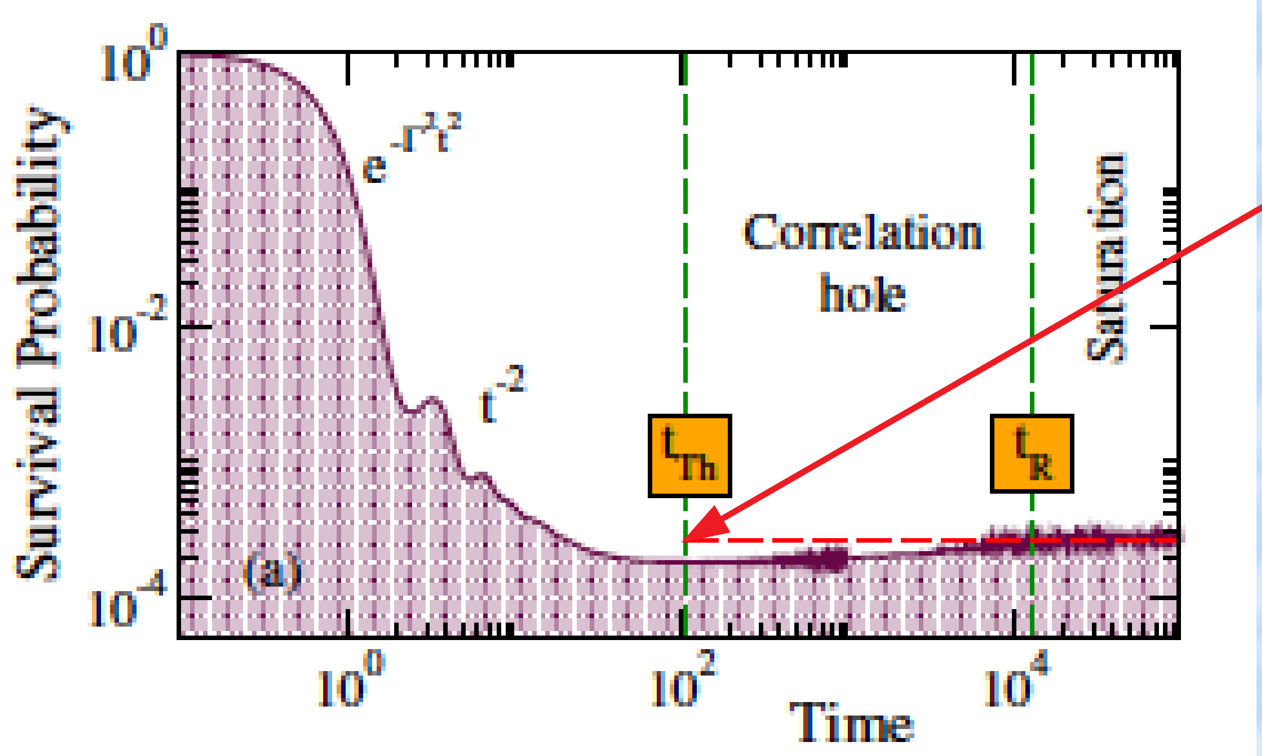

Step two: the observable quantity reaches a value smaller than the asymptotic one (CORRELATION HOLE)

Time scale: Thouless time t_{Th}

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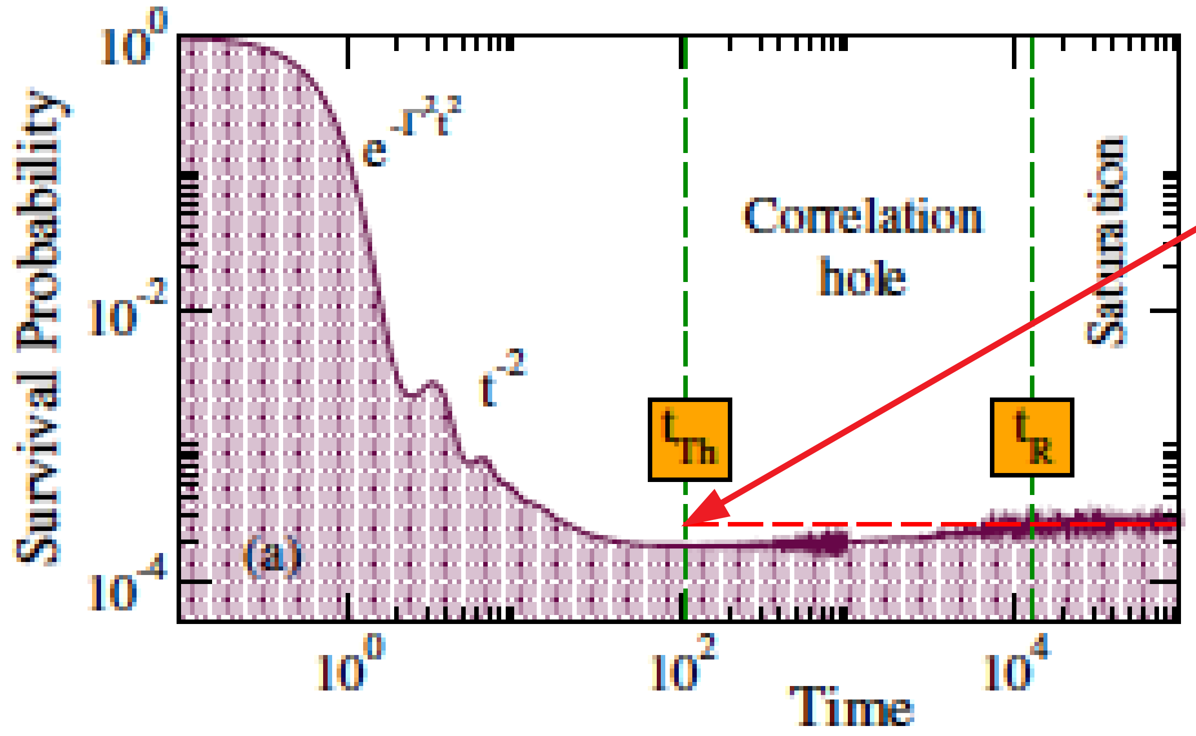

Step three: saturation to the asymptotic value

Time scale: relaxation time t_R

GOE MATRICES: SURVIVAL PROBABILITY

GOE: ensamble of matrices with real, independent, Gaussian entries.

We compute the Survival Probability P_S of the initial state

$$
\begin{aligned}
P_S(t) &= \left|\langle\psi_0|\, e^{-iHt}\, |\psi_0\rangle\right|^2 \\
&= \int dE e^{-iEt} \rho_0(E) \\
&= \int dE e^{-iEt} \sum_{\alpha_1 \neq \alpha_2} |c^0_{\alpha_1}|^2 |c^0_{\alpha_2}|^2 \delta(E - E_{\alpha_1} - E_{\alpha_2}) + P_S(\infty)
\end{aligned}
$$

GOE MATRICES: SURVIVAL PROBABILITY

The ensemble average for P_S can be computed analytically!

$$P_S(t) = \frac{1-\overline{P_S}}{D-1}\left[D\frac{\mathcal{J}_1^2(2\Gamma t)}{(\Gamma t)^2} - b_2\left(\frac{\Gamma t}{2D}\right)\right] + \overline{P_S}$$

Fourier transform of the LDOS for infinite systems: approximates the spectrum as a continuum!

Fourier transform of the level-level correlation function: takes into account the discreteness!

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$$\frac{\mathcal{J}_1^2(t)}{t^2} \sim \begin{cases} 1-t^2 & t \ll 1 \\ \frac{1}{t^3} & t \gg 1 \end{cases}$$

Fourier transform of the level-level correlation function: takes into account the discreteness!

$$b_2(t) \sim \begin{cases} 1-2t & t \ll 1 \\ \frac{1}{t^2} & t \gg 1 \end{cases}$$

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D dimension of the Hilbert space

Γ width of the LDOS

$$\Gamma = \sqrt{D}$$

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$$\Gamma = \sqrt{D}$$

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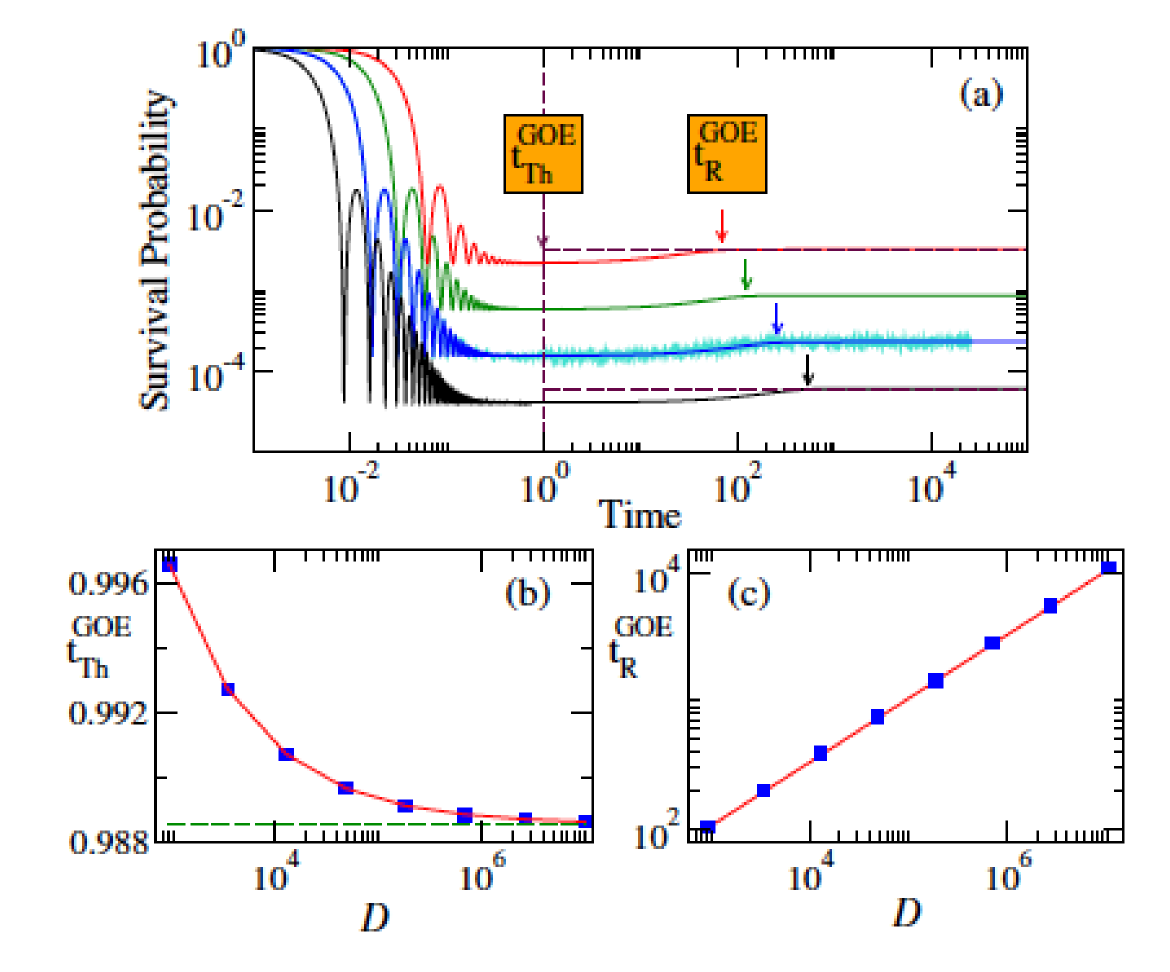
Position of the minimum:

$$t_{\mathrm{Th}} = \left(\frac{3}{\pi}\right)^{\frac{1}{4}} \frac{\sqrt{D}}{\Gamma} = \left(\frac{3}{\pi}\right)^{\frac{1}{4}}$$

Saturation:

$$t_R \propto \frac{D}{\Gamma} = \sqrt{D}$$

GOE MATRICES: SURVIVAL PROBABILITY



SPIN CHAINS: SURVIVAL PROBABILITY

Disordered Heisenberg model

$$H = J\sum_{i=1}^{L} \vec{S}_i \cdot \vec{S}_{i+1} + \sum_{i=1}^{L} h_i S_i^z$$

To compute P_S: repeat the calculation, using the (Gaussian) shape of the LDOS

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$$P_S(t) = \frac{1-\overline{P_S}}{D-1}\left[D\frac{e^{-\Gamma^2 t^2}}{(\Gamma t)^2}g(t) - b_2\left(\frac{\Gamma t}{2D}\right)\right] + \overline{P_S}$$

$$g(t) = [(\Gamma t)^2 + A(e^{-\Gamma^2 t^2} - 1)]/(1+A)$$

This time, the Hamiltonian matrix is sparse:

$$\Gamma \propto \sqrt{L}$$

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$$P_S(t) = \frac{1-\overline{P_S}}{D-1}\left[D\frac{e^{-\Gamma^2 t^2}}{(\Gamma t)^2}g(t) - b_2\left(\frac{\Gamma t}{2D}\right)\right] + \overline{P_S}$$

$$\Gamma \propto \sqrt{L}$$

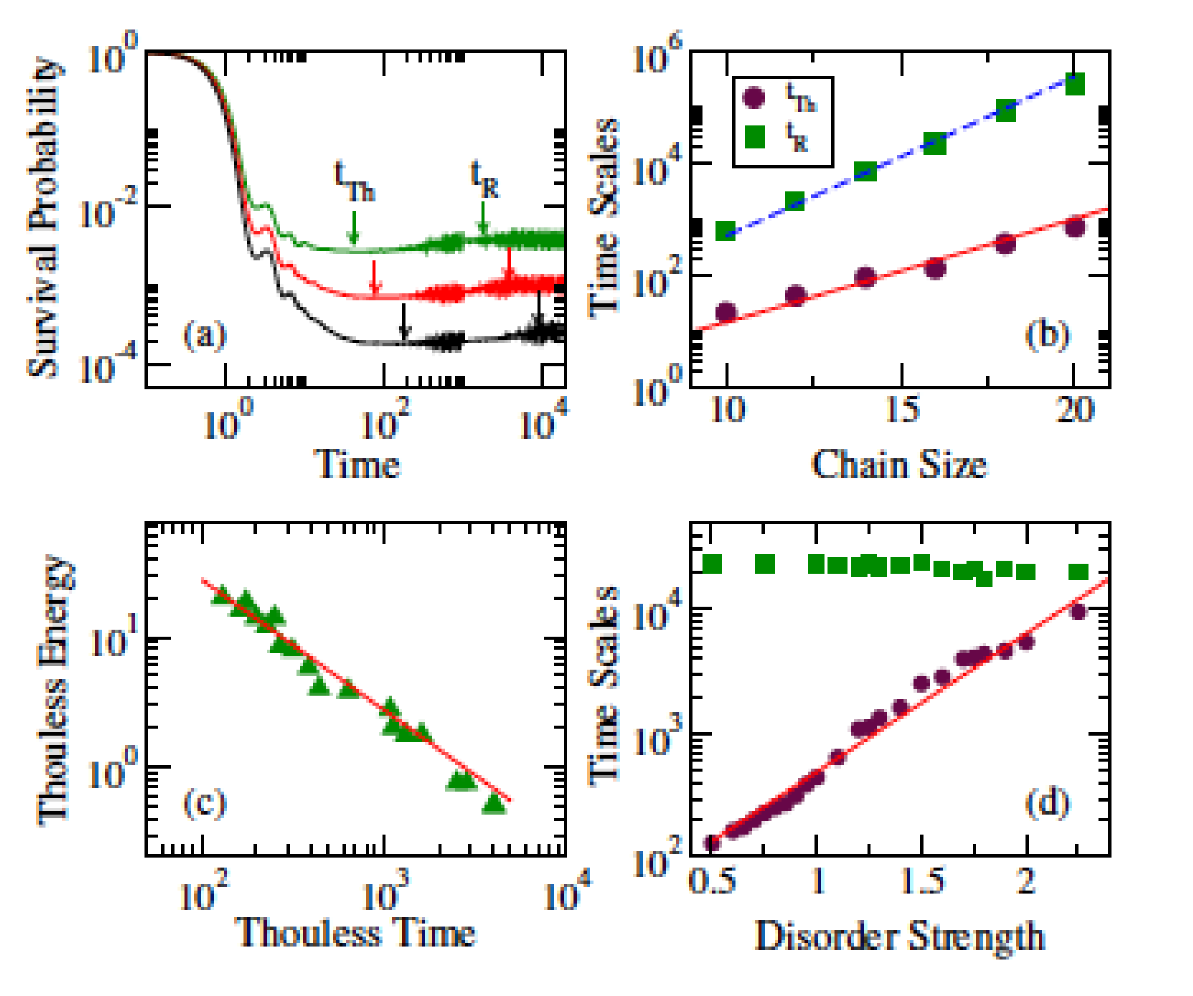
Position of the minimum:

$$t_{\mathrm{Th}} \propto \frac{D^{\frac{2}{3}}}{\Gamma} \sim \frac{e^{2\ln(2)/3}}{\sqrt{L}}$$

Saturation:

$$t_R \propto \frac{D}{\Gamma} = \frac{D}{\sqrt{L}}$$

SPIN CHAINS: SURVIVAL PROBABILITY



SPIN CHAINS: SPIN AUTOCORRELATOR

Local operator, easier to measure in real experiments:

$$I(t) = \frac{4}{L} \sum_{i=1}^{L} \langle \psi_0 | \, S_i^z e^{iHt} S_i^z e^{-iHt} \, | \psi_0 \rangle$$

No analytic results, but numerically found to behave qualitatively analogously to P_S!

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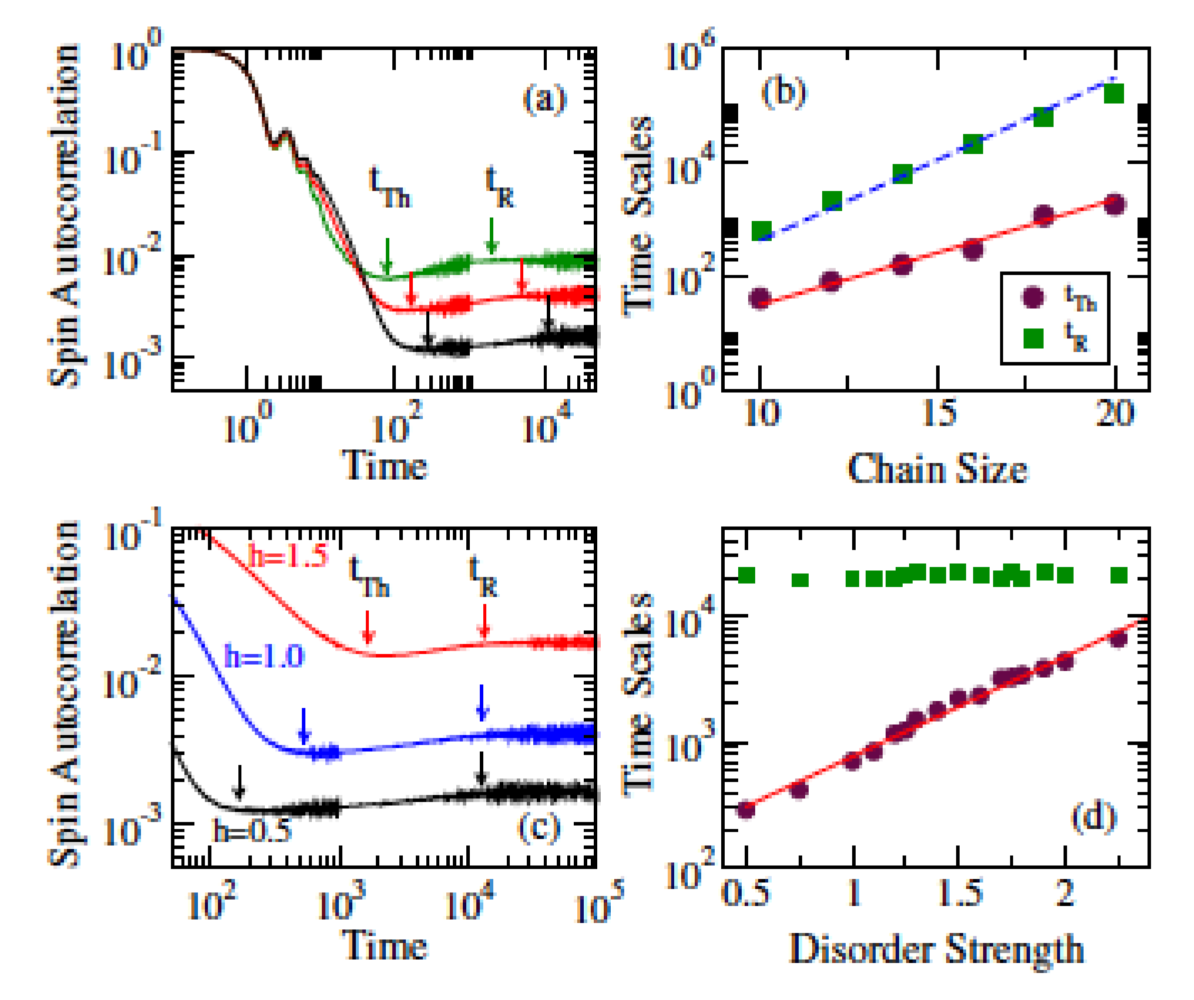

CONCLUSIONS

- Two time scales, both exponentially long for physical systems
- They show up for both local and non local quantities
- Can be studied analytically, using input from random matrix theory